



Systems
Optimization
Laboratory

LEVEL





DOC FILE COPY

THE DOCUMENT IS REST QUALITY PRACTICAL THE COPY FUNDISHED TO DISC CONTAINED A STORIFICART MOVER OF PAGES WEIGH DO BOX REPRODUCE LEGIBLE.

This document has been approved for public release and sale; its distribution is unlimited.

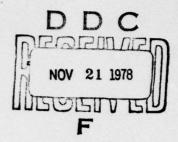
Department of Operations Research Stanford University Stanford, CA 94305

78 11 16 002

DISCLAIMER NOTICE

THIS DOCUMENT IS BEST QUALITY PRACTICABLE. THE COPY FURNISHED TO DDC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

SYSTEMS OPTIMIZATION LABORATORY
DEPARTMENT OF OPERATIONS RESEARCH
Stanford University
Stanford, California
94305



BCA AND HRA: TWO PROGRAMS FOR COMPUTING ECONOMIC EQUILIBRIA

by

Thomas Elken

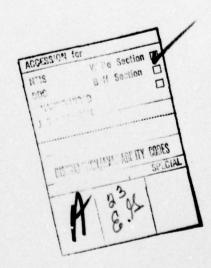
TECHNICAL REPORT SOL 78-17 August 1978

Research and reproduction of this report were partially supported by the U.S. Department of Energy Contract EY-76-S-03-0326 PA #18; The Office of Naval Research Contract NO0014-75-C-0267; and the National Science Foundation Grants MCS76-20019 A01 and MCS76-81259 A01.

Reproduction in whole or in part is permitted for any purposes of the United States Government. This document has been approved for public release and sale; its distribution is unlimited.

TABLE OF CONTENTS

Section		Page
I	Introduction	1
II	The Bilinear Complementarity Algorithm (BCA)	7
	II.1. Input Requirements	
	II.3. Subroutines of BCA	21
	II.4. Sample Problems	
III	The HRA (Homotopy Retraction Algorithm) Code	
	for Solving Equilibrium Problems	79
	III.1. Revisions to the Original Code	
	III.2. Input Requirements	
	III.3. Main Program	
	III.4. Subroutines of HRA	
	III.5. Sample Problems	89
	III.6. HRA Source Listing	96
	References	128



BCA and HRA: Two Programs for Computing Economic Equilibria

by

Thomas Elken

I. Introduction

BCA and HRA are computer programs designed to solve a version of the economic equilibrium problem. For an introduction to this type of problem and models for dealing with it, the reader is directed to Elken [2]. Very briefly, we will describe mathematically the problem which these codes solve.

Find x, t, λ , and ζ such that

$$Dx + t = b$$

$$\lambda D - \zeta = e_{NCOL}$$

$$(c_i, \lambda) - \lambda_i t_i = 0$$
, $i = 1, ..., IH$ (1)

$$\lambda_{i}t_{i} = 0$$
 , $i = 1H + 1, ..., NROW$

 $x, t, \lambda, \zeta \geq 0$,

where D is a matrix with NROW rows and NCOL columns, IH is the number of consumers or households, e_{NCOL} is the NCOLth unit vector, and $\langle \cdot, \cdot \rangle$ is the usual inner product.

The BCA is an implementation of the bilinear complementarity algorithm presented in Wilson [7] and in Elken [2]. The HRA code implements the homotopy retraction algorithm as described in [2]. The reader must consult the latter work to be able to formulate a problem correctly and to be cognizant of the conditions under which these algorithms will solve his/her problem.

The algorithms have so many features in common that many of the subroutines in the two programs are identical. In particular, both algorithms begin by solving the linear program

maximize x_{NROW}

subject to
$$[I|D] \left[\frac{t}{x}\right] = b$$
,

(2)

t, $x \ge 0$.

To solve this linear program, the code LPM1 written by John Tomlin is used. Actually, the subroutines comprising LPM1 are part of both codes, BCA and HRA. LPM1 has been documented in J. Tomlin [4]. LPM1 has been converted into a linear complementarity code, LCPL, which has been documented in J. Tomlin [5] and [6]. Since

LCPL uses many of the subroutines of LPM1 without change, the reader is referred to the works cited for a description of how the matrix A = [I|D] is stored and how transformations are processed.

Next we discuss some important details of the implementation of BCA and HRA.

Source Language

The programs are written entirely in FORTRAN IV for the IBM series 360 and 370 computers. They are WATFIV compatible.

Specification

The main program is executed as a job step. The program input is described in Section II.1. All input occurs in subroutines MAIN, INPUT, and BASINP.

Error Indicators

Error indicators and other diagnostics and messages will be indicated in the documentation for each subroutine. The error indicators for the LPM1 portion of the code are described in [4].

Subroutines

The routines making up BCA and HRA are as follows:

BCA									HRA
MAIN									MAIN
BLCONS									BLCONS
FINDP									FINDP
PIVOT									PIVOT
UPAKC									UPAKC
BSCNG									BSCNG
SUPERB									SUPERB
RECALC									RECALC
ENDPNT									ENDPNT
QUADS									QUADS
DSENT									DSENT
									GTN
									DERIVG
CONCHK									CONCHK
FTN									FTN
DERIV									DERIV
NORM									NORM
DECOMP									DECOMP
SOLVE									SOLVE
SING									SING
DEBUG									DEBUG

The dotted lines indicate the subroutines which are essentially identical.

The remainder of the subroutines are from LPM1 for both programs.

BLOCK DATA

INPUT

FTRAN
BTRAN
FORMC
TRUEDJ
PRICE

CHSOL
UPBETA
NORMAL
ITEROP
UNRAVL
CRASH
BASINP

CLEAR

SHIFTR INVERT

CHUZR

WRETA

UNPACK

SHFTE

These subroutines are described in greater detail in the remaining sections of this report.

Program Size

The total length of BCA is 3831 source statements (including comments and common blocks, etc.); HRA is 3772 statements long.

LPM1 comprises 1819 statements of each of these.

Array Size

The standard version of the codes requires a blank common array of 133,256 bytes of core. This is dependent on the setting of maximum problem dimensions.

There are eleven labelled common blocks which require 97,448 bytes of core. Only the size of common blocks LNCONS and INDXZ are dependent upon problem dimensions.

Accuracy and Convergence

The linear constraints (1) and (2) are satisfied with a tolerance of ZTOLZE with a default of 1.E - 4. The bilinear constraints (3) are satisfied to a tolerance of TOLFZ; 1.E - 10 is the default.

Convergence is guaranteed in the BCA and HRA when the first

IH values of b are chosen appropriately. See Elken [2] Chapters

V-VI for a discussion of the convergence properties of these algorithms and suggestions for improvements.

Timing

Using the FORTRAN H compiler with OPT = 2 the codes have solved a problem with 6 consumers and 4 goods with $D \in R^{23\times20}$ in .45 and .38 seconds for the BCA and HRA, respectively. A problem with 3 consumers and 56 goods and $D \in R^{97\times107}$ required 4.54 and 6.19 seconds for the BCA and HRA to solve. A description of numerical results for these and several more test problems is in Chapter V of [2].

II. The Bilinear Complementarity Algorithm (BCA)

II.1. Input Requirements

The program input consists of 4 (or 5) segments.

- A) Parameter specification relevant to equilibrium problem
- B) Parameter specification relevant to the auxiliary linear program
- C) Data for the LP in MPS standard form
- D) Basis input (optional, see [5])
- E) Specification of the C matrix (3). (Initial endowments for the individual consumers.)

We will now deal with these in succession.

A. Program Parameter Input

Parameters are changed from their default value for each problem by means of a FORTRAN "NAMELIST" input from the card reader. The first card must start with &PARM1 in column 2 or later; each card must begin with a blank, and the list must be terminated by &END.

Example

columns

1 2 3

&PARM1 IH = 3, L = 2, ICECHO = 1, &END

The parameters which can be defined by the &PARM1 statement are listed below along with their default values.

REAL *4 Tolerances

TOLFZ (DEFAULT = 1.E - 10)

Termination criterion for the solution of the bilinear equations.

TOLBD (DEFAULT = 1.E - 4)

Feasibility tolerance for the linear constraints.

TOLCV (DEFAULT = 1.E - 5)

Termination criterion for the return to the curve.

INTEGER *4 Parameters

IH Number of consumers (traders). This parameter has no default.

It <u>must</u> be specified in the &PARM1 statement.

ICNTRL (DEFAULT = 1)

If ICNTRL.LT.O, then the program behaves just like LPM1 to solve one or more linear programs. This may be useful when one desires to solve some preliminary linear programs to determine initial utility levels. Otherwise, an equilibrium problem will be processed by BCA.

IECHO (DEFAULT = 0)

If IECHO = 1 while ICNTRL.LT.O, the D matrix of (1)
will be printed out, ten columns at a time.

ICECHO (DEFAULT = 0)

If .EQ.1, the C-matrix of (3) will be printed.

IPROD (DEFAULT = 0)

If .EQ.1, it is assumed that the consumers own certain shares of the producing firms. Thus, data containing those shares must appear following the LP data in 10F7.4 format.

ITLIME (DEFAULT = 500)

The maximum number of cells which we allow the program to pass through.

KOUTB (DEFAULT = 0)

Unit number of output of the optimal basis for the auxiliary linear program. If KOUTB.EQ.O, no basis will be saved.

REAL *4 Parameters

STPMX (DEFAULT = 100)

The maximum possible step in the direction tangent to the curve.

STPRD (DEFAULT = 0.5)

The factor between $\,0\,$ and $\,1\,$ used to reduce the steplength when it exceeds STPMX.

B. Linear Program Parameters

Next, a list containing parameters of importance for the linear programming portion of the code must be included. The beginning of this list is &PARAM. We leave the description of these parameters to [4] with one exception. The parameter IOBJ contains the row number of the objective row in the D matrix. Since this row is always e_{NROW}^T (2) no flexibility is necessary here so we require that IOBJ be the last row of D. Since we did not want to alter the LPM1 part of the code, the user must know how many rows D has and enter the appropriate value for IOBJ in the parameter list. This rather irritating requirement makes bookkeeping easier in many parts of the code.

Also, the auxiliary linear program (4) is a maximization problem while LPM1 is a minimization code by default. This can be remedied in one of three ways:

- 1) Let the last row of D be $-e_{NROW}^{T}$,
- 2) Set ZSCALE =-1.0 in the &PARAM list, or
- 3) Set IOBJ = -NROW in the &PARAM list.

Remember to do only one of the alternatives above.

C. Problem Input

The D and b referred to in (1) are read from unit KINP (DEFAULT = 5) in slightly modified "MPS format". The card images required are:

NAME Card

This has "NAME" in columns 1 to 4 and the (up to 8 characters) problem name in columns 15-22. This card is optional but highly desirable.

ROWS Card

Has "ROWS" in columns 1-4.

Row Names

Each row is assigned a (unique) name of up to 8 characters, one per card, in columns 5-12. Embedded blanks are allowed (unlike

MPS). Also row types must be supplied in columns 3 of each card

L: < constraint

G: ≥ constraint

E: = constraint

N: objective function row .

For the equilibrium problem all constraints will be of type "L" except for the objective function.

COLUMNS Card

Has "COLUMNS" in columns 1-7.

Matrix-Element

The non-zero elements of D are supplied by column. All the elements of a column must be together. Each column is assigned a (unique) name of up to eight characters. The format is:

cols 1-4 5-12 15-22 25-36 40-47 50-61 blank column row element second second name name value row element name value (optional) (optional)

Again embedded blanks are allowed in column names.

RHS Card

Has "RHS" in columns 1-3.

Right Hand Side Elements

The right hand side vector (b) may be given a name, which should be different from any row or column name. The elements are given in the same format as the matrix elements.

ENDATA Card

Has "ENDATA" in columns 1-6. Sample input is shown in Section II.4.

D. LP Basis Input

If KINB is initialized to a nonzero value in &PARAM, a basis is read from unit KINB. If KINB.EQ.KINP, the basis must be entered following the LP problem input, and before the ENDATA card. This basis will be the initial basis for solving the auxiliary LP (2).

NAME Card

Has "NAME" in columns 1-4 and the basis name in columns
15-22. If this name does not agree with the problem name, or the name
card is missing, a warning is given.

Basis Cards

For basic columns of D, the column names are given (one per card) in columns 5-12. For basic columns of I (slack variables), the corresponding row name is given in columns 5-12.

ENDATA Card

Has ENDATA in columns 1-6.

E. C Matrix Input

In most problems the data for C will be predominately extracted from the RHS vector b. For this reason we use a rather short, but complicated, form for the input of C.

The data is read from unit KINP directly following the MPS format data. The instructions constructed from the pointers in this data produce a matrix packed into a vector in the same manner that [I|D] is packed (see [6]).

In the current version of the code it is assumed that (if IPROD.NE.0) each consumer I is associated with a fraction SHR(I), which represents his share of each firm in the economy, such that $\sum_{I=1}^{IH} \text{SHR}(I) = 1.0.$ If this assumption is not true, the correct C matrix can still be input; however, it requires more work.

The form of the data for inputting the C matrix is as follows

 One or more cards containing (SHR(I), I = 1, IH) in 10F7.4 format (only if IPROD.NE.O).

- Several cards describing the column of C associated with consumer 1.
- 3. A blank card.
- 4. Information associated with consumer 2, etc.

We now elaborate on the specific requirements of point 2 above. Assume that we are dealing with column $\, { ext{I}} \,$ of $\, { ext{C}} \, .$

The first card for column I contains values for the integer variables ISHR, LL, and KK in 3I4 format. ISHR is a control parameter which behaves as follows:

- A) If ISHR.EQ.1, the program reads one or more consecutive elements of the RHS corresponding to constraints on one or more of the firms in the economy. These numbers are multiplied by SHR(I), the Ith consumers share of the firm, and stored in C.
- B) If ISHR.EQ.O, the program reads one or more consecutive elements of the RHS corresponding to the constraints on consumer I alone.
- C) If ISHR.EQ.-1, the program reads one or more elements from the card(s) following in the input stream in 10F7.4 format. These usually correspond to commodities (or firms) which are owned by a number of consumers, but not in any fixed proportion.

LL and KK determine the range of rows of C which will be altered due to this card. If only one row is involved, either KK can be set equal to zero (leave columns 9-12 blank), or set KK equal to LL.

A blank card indicates that the program should move on to the next column of C (the next consumer).

If the problem has been formulated and input correctly, the sum of the columns of C must be equal to the RHS vector (see [2], Section V.3). Examples of input and output for some small problems will be given after the program in Section II.4.

II.2. Main Program

The main program consists of 350 statements which perform a variety of functions: the parameters are initialized and altered by reading the name list PARMI, the subroutines of LPMI are called to solve the auxiliary linear program, the data is read so that the C matrix can be constructed, the machinery is set up for the bilinear complementarity algorithm to begin, the main path-following subroutine ENDPNT is called, the updates and decisions made necessary by the output from ENDPNT are performed, and the final solution is printed. If options were added so that a number of equilibrium problems could be solved, this main program would undoubtedly become sort of a master subroutine, as NORMAL is for LPMI.

Restrictions relevant to the use of BCA.

- 1. The number of consumers (IH) must be less than or equal to 10.
- 2. A must have not more than 350 rows or 400 columns.

Of course these limits can be extended by redimensioning all vectors of size 10, 350 or 400 and allocating more core.

Before describing the subroutines of BCA we will define the variables in the labelled common blocks and some of those in blank common (those of use in the subroutines outside of LPM1).

Variable Glossary

We have specified that almost all real variables are REAL*8 by the statement

IMPLICIT REAL*8 (A-H, O-Z) .

Notable exceptions are the initial data in A and C which are stored as REAL*4 variables.

COMMON/LP1/:

- PI(1302) The complete set of dual variables (prices and relative costs).
- XX(1302) The complete set of primal variables, slacks first and then the structural variables in their original order.

COMMON/BLCST/ (Bilinear constraints)

COMMON/BLCST2/

These variables store the current version of the budget surplus function in terms of the superbasic variables (see Elken [2] Section III.3). The functional form is

Type 1:
$$f_{\mu} = BF1 + D1*PI(MU) - DIAG(PI(MU))*(F1*XX(NU) + E1)$$
(3)

Type 2:
$$f_v = BF2 + D2**PI(MU) - DIAG(XX(NU))*(F2*PI(MU) + E2)$$
,

where PI(MU) refers to those components PI(J) of PI such that MU(I) = J for some I = 1, 2, ..., KMU, KMU is the current number of dual superbasic variables, and

$$DIAG(PI(MU)) = \begin{pmatrix} PI(MU(1)) & 0 & \cdots & 0 \\ 0 & PI(MU(2)) & 0 \\ \vdots & & \ddots & \\ 0 & & & PI(MU(KMU) \end{pmatrix}.$$

XX(NU) and KMU are defined similarly.

Also in COMMON/BLCST/ are C(800), IC(800), and LC(20) which store C in packed form in exactly the same manner as A.

COMMON/LNCONS/

These matrices and vectors allow us to express the basic variables in terms of the superbasic variables. If IH and DIGMA are index sets for the primal and dual variables, respectively, then the relationship is

$$XX(JH(I)) = \sum_{J=1}^{KNU} G1(I,J)*XX(NU(J) + BA(I)$$
 $i = 1, ..., M$

$$PI(DIGMA(I)) = \sum_{J=1}^{KMU} G2(I,J) *PI(MU(J)) + BB(I)$$
 $I = 1, ..., N$,

where the arrays are dimensioned G1(350,10), G2(400,10), BA(350), BB(400).

COMMON/INDX1/NUH(10), MUH(10), NU(10), MU(10)

$$NUH(I) = \begin{cases} 0 & \text{if } XX(I) & \text{is not superbasic} \\ \\ \\ J & \text{if } XX(I) & \text{is the Jth} \end{cases}$$

superbasic variable I = 1, ..., IH

MUH(I) =
$$\begin{cases}
0 & \text{if PI(I) is not superbasic} \\
& \\
J & \text{is PI(I) is the Jth}
\end{cases}$$

dual superbasic variable, I = 1, ..., IH

NU(I) = J if the Ith superbasic variable is XX(J)

MU(I) = J if the Ith dual superbasic variable is PI(J).

Note that NUH(NU(I)) = I.

COMMON/INDX2/JH, DIGMA, KINBAS, IDBAS

These arrays correcpond to $\sigma, \bar{\sigma}, \gamma$ and $\bar{\gamma}$ in the description of the algorithm in Chapter III of [2].

JH(I) = J if XX(J) is basic and pivots on row I.

DIGMA(I) = J if PI(J) is dual basic and corresponds to the Ith
 row of G2.

 $IDBAS(I) = \begin{cases} 0 & \text{if } PI(I) \text{ is not dual basic} \\ \\ j & \text{if } PI(I) \text{ is dual basic and corresponds to the Jth} \end{cases}$ row of G2.

COMMON/SCAL/

BT = A scalar that helps to define the normal hyperplane subproblem in ENDPNT.

JJ = The index of the constraint in G1 or G2 which defines the exiting facet for the current cell.

MFLAG = A flag which defines the type of jacobian which must be calculated.

DD1 = The number of bilinear constraints which are currently at zero.

P = The index of the pivot column determined by subroutine FINDP.

PD = The flag which determines whether the initial facet corresponds
to a primal (PD = 1), a dual (PD = -1) variable, or a bilinear
constraint (PD = 0) at zero.

1, if the bilinear inequality is the last of the Type 1 constraints.

INEQ =

2, if the bilinear inequality is the last of the Type 2 constraints see (1).

KFUN = Counts the number of functionals evaluated in (1).

KJAC = The number of partial derivatives of the functions defined by (1) which are calculated during the course of the algorithm.

COMMON/DIM/ (DIMENSIONS)

IH = The number of households.

M = Number of rows in the problem.

N = Number of structural variables in the problem.

DD = Current number of superbasic variables (= DD1 + 1).

KMU = Current number of primal superbasics.

KNU = Current number of dual superbasics (DD = KMU + KNU)

BL1 =
$$\begin{cases} KMU - 1, & \text{if INEQ} = 1 \\ \\ KMU, & \text{otherwise} \end{cases}$$

BL2 =
$$\begin{cases} KNU - 1, & \text{if } INEQ = 2 \\ \\ KNU, & \text{otherwise} \end{cases}$$

MP1 = M + 1

NM = N + M

COMMON/INT/

IPS(30) A vector storing the permutations for a LU decomposition linear equation solver used in conjunction with Newton's method.

KDET The sign of the determinant of the current jacobian of f.

KOUNT The number of times ENDPNT has been called.

ISING A flag which is 1 when the current jacobian is singular,

0 otherwise.

COMMON/TOLER/TOLFZ, TOLBD, TOLCV, ICNTRL, IECHO
These parameters are defined above.

II.3. Subroutines of BCA

(2) SUBROUTINE BLCONS: Calculates the coefficients of the first DD bilinear constraints. DDl of these are constraints which define the curve. The last is an inequality which is not yet binding, but which must be the next one to become binding.

The calculations are motivated by the fact that the bilinear functions

$$\sum_{j=1}^{M} C(I,J)*PI(J) - PI(I)*XX(I), I = 1, ..., DD$$

can be reduced to functions of the superbasic variables by using (6). The position of DD in the index sets MU and NU determines BL1, BL2, INP, INQ, and INEQ.

- (3) SUBROUTINE FINDP (PD1, IS, P): Finds the element of largest absolute value in the row determined by (PD1, IS) and stores the index of the variable corresponding to that element in P. When we say the row determined by (PD1, IS) we mean G1(IS, ·) if PD1 = 1, and G2(IS, ·) if PD1 = -1.
- (4) SUBROUTINE PIVOT (SS,RR): If variable XX(SS) is entering the basis and variable XX(RR) is leaving the basis, the columns of Gl and G2 and the columns BA and BB must be updated. This is accomplished by a simple pivot. The details for this pivot are contained in Section III.5 of [2].
- (5) SUBROUTINE UNPACKC(II): A call to this subroutine causes the

 IIth column of C to be unpacked and stored in the M-vector Y.
- (6) SUBROUTINE BSCNG(S,R): This subroutine updates the index sets

 JH, KINBAS, DIGMA, and IDBAS when variable S replaces variable

 R in the primal basis.
- (7) SUBROUTINE SUPERB(KEY, PD1, IS, PD2,JS): Revises the index sets defining the superbasic variables. It also adds or removes columns of G1 and G2 depending on the values of the parameters

KEY = $\begin{cases} 0, & \text{if columns are both added and removed,} \\ 1, & \text{if columns are only added,} \\ 2, & \text{if columns are only removed.} \end{cases}$

(PD1, IS) determine the column to be added.

(PD2, JS) determine the column to be removed.

The calculations involved in calculating the column to be added are described in [2] Section III.3.

- (8) SUBROUTINE RECALC: The parameter INVFRQ of LPM1 specifies how frequently the product form basis is to be reinverted. Each time the basis is reinverted while the BCA portion of the code is in effect, G1, G2, and BA, BB are recomputed using the new basis inverse. The same thing could be accomplished by DD calls to SUPERB with KEY = 1, and (PD1, IS) appropriately specified.
- (9) SUBROUTINE ENDPNT (JS, PD1, IS, NET): implements the path-following algorithm described in Section III.4 [2]. The initial point is in a facet of the cell defined by (6) and the bilinear inequality; we call this the initial facet. The algorithm follows the curve defined by the DD1 bilinear constraints and the DD superbasic variables. The objective is to follow the curve until it intersects another facet of the cell and to find the point where that intersection occurs. We call that point the endpoint which is contained in the final facet. The initial facet is determined by the equation described by (PD1, JS). The final facet is determined by (PD1, IS). NET counts the number of Newton iterations which were required in the various subproblems of the algorithms referred to above.

A common block is referred to for the first time in this sub-routine: COMMON/NEWT/H(10, 11), X(10) Z(10), ACC(2, 10). The following vectors are used in this routine: U(3), V1(10), (V2(10) F(10), DOT(4), RHS(10), UL(10, 10).

- H(10, 11) stores the jacobian for the various Newton subproblems
- X(10) contains the superbasic variables as X = (PI(MU), XX(NU))
- Z(10) is the correction to X supplied by the Newton method, $Z = H^{-1}F$
- ACC(2, 10) sotres the two vectors which determine the current linear approximation to the curve. $f^{-1}(0)$
- U(3) stores the coefficients for any quadratics that have to be solved
- V1(10) holds the gradient of the functional defining the initial facet
- V2(10) contains a vector in the null space of H, a tangent to $f^{-1}(0)$
- F(10) holds the values of f and any other functional involved in the current subproblem
- DOT(4) a collection of accumulators
- RHS(10) a work space for finding the tangent vector by solving a linear system; RHS = $H(\cdot, ICX)$.
- UL(10, 10) stores the LU decomposition of the current jacobian matrix

Subroutines called: QUADS, DSENT, CONCHK, FTN, DERIV NORM, DECOMP, SOLVE.

It should be noted that the Newton's method being implemented is a modified Newton's method with descent. The iteration is

$$X^{k+1} = X^k - \alpha(f'(x^k)^{-1} f(x^k))$$
 $k = 0, 1, ...$

where α is the first scalar of $\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\}$ such that

$$\|f(x^{k+1})\| < \|f(x^k)\|$$
.

The basic loop is something like the following

DO 100 K = 1, 15

CALL FTN(F, X)

CALL NORM (F, S1, DD)

IF (S1.LT.TOLCV) GO to 800

CALL DERIV

CALL DECOMP (DD, H, UL)

CALL SOLVE(DD, UL, F, Z)

ALPHA = 1.0

CALL DSENT (ALPHA, S1)

DO 95 I = 1, DD

X(I) = X(I) - ALPHA*Z(I)

95 CONTINUE

100 CONTINUE

C TAKE CORRECTIVE ACTION IF NEWTONS

C METHOD HAS NOT TERMINATED WITHIN

C 15 ITERATIONS

calculate f(X)

set S1: = ||f(X)||

terminate if $S1 < 10^{-5}$

calculate H: = f'(X)

decompose H = UL

solve by back substitution

UL *Z = F

implements descent on the

norm of f.

(10) SUBROUTINE QUADS (U, IMAG, ALPHA, BETA): finds both real roots of the quadratic

$$U(1) \cdot \alpha^2 + U(2) \cdot \alpha + U(3) = 0$$
.

If there are no real roots, the flag IMAG is set equal to 1. If there is a double root or if U(1) = 0, the root is stored in both ALPHA and BETA. If there are two real roots ALPHA stores the smaller root and BETA the larger.

- (11) SUBROUTINE CONCHK (GMIN, KGMIN, MP2): checks whether the constraints of (1) are satisfied at the point defined by the current superbasic values. The basic variables are evaluated in terms of the superbasic variables as suggested by (4). GMIN stores the minimum value of these variables and (MP2, KGMIN) indicate which variable attains this minimum. The theory tells us that we only have to check the current bilinear inequality among the bilinear constraints described in (3). If this value is less than GMIN, MP2 is set equal to zero.
- (12) SUBROUTINE FTN(F, X): evaluates the first DD1 bilinear functionals as they were defined in BLCONS (3). The last component F(DD) stores a different function value depending upon which subproblem is being solved in ENDPNT.

If MFLAG = 0, then F(DD) = 0.

- If MFLAG = 1, then F(DD) contains the value of the functional specified by (MPD, JJ).
- If MFLAG = 2, then F(DD) = X(JJ).
- If MFLAG = 3, then $F(DD) = \langle ACC(2, \cdot), X \rangle BT$ (the normal hyperplane to $ACC(2, \cdot)$, if F(DD) = 0).
- (13) SUBROUTINE DERIV: computes the exact jacobian of the function calculated in f. The form of the jacobian is as follows

$$H = \begin{bmatrix} D1 & DIAG(PI(MU))*F1 \\ D2 & -DIAG(F2*(PI(MU) + E2) \end{bmatrix} - \begin{bmatrix} DIAG(F1*XX(NU) + E1) & 0 \\ DIAG(XX(NU))*F2 & 0 \end{bmatrix}$$

The last row of H contains the gradient of the last functional of F as described above.

- (14) SUBROUTINE DSENT (ALPHA, PNORM): Implements descent on the norm of f in Newton's method as described above. ALPHA is cut in half until $\|f(x^{k+1})\| < PNORM = \|f(x^k)\|$ or until ALPHA $\leq 10^{-5}$.
- (15) SUBROUTINE DECOMP (NN, A, UL): (Borrowed from Forsythe [3].

 The reader should look there for a complete description.)

 Implements a Gaussian elimination scheme with partial pivoting to produce a LU decomposition of the matrix A. A few statements were added to this subroutine so that KDET would change sign every time a row interchange was performed. KDET will eventually be the sign of det A (see Elken [1977a], sec. III.2).

(16) SUBROUTINE SOLVE (NN, UL, B, X): is also borrowed from Forsythe [3] with the additions of statements which change the sign of KDET every time UL(I, I) < 0. SOLVE uses backsubstitution to solve the system L*U*X = B. L and U are a lower and upper triangular matrix which are stored in the square matrix UL.

At the end of SOLVE, KDET is equal to sgn(det A).

- (17) SUBROUTINE SING(IWHY): is also borrowed from [3]. It is called from DECOMP and SOLVE to print messages concerning the singularity of A depending on the parameter IWHY.
- (18) SUBROUTINE DEBUG(MODE): prints information which may be useful in debugging the program when changes are made. The information which is printed depends upon the value of MODE.
 - MODE = 1: Prints the time left in this job step by calling the system subroutine LEFTLA(TIME) where TIME is a REAL*4 variable. When running this program in WATFIV or on a system other than SLAC, this portion of code should be removed.
 - MODE = 2: The arrays BA, Gl, BB, and G2 will be printed by rows.
 - MODE = 3: The index sets defining the primal basis, JH and KINBAS will be printed.
 - MODE = 4: Information about the location and value of the first IH primal and dual variables will be given (KNU, KMU, NVH, MUH, (XX(I), PI(I), I = 1, ..., IH)).

- MODE = 5: The coefficients of the bilinear constraints will be printed: BF1, BF2, D1, D2, E1, E2, F1, and F2.
- MODE = 6: A variety of variables will be printed out which are of use in debugging ENDPNT: PD, KMU, KNU, INEQ, MUH, NUH, JH, DIGMA, PI and XX.

The remaining 1815 lines of BCA contain the subroutines which comprise LPM1. The reader is referred to [4], [5], and [6] for documentation relating to them.

II.4. Sample Problems

The sample input and output are given below for two small problems. The computational results for these problems are given in [2] Chapter V.

For the reader to learn how to formulate an equilibrium problem so that it can be input and solved by BCA, (or HRA) it is necessary to read Sections II.2 and II.3 of [2]. We will discuss the definitions of D, b and C of (1) for a small problem here.

The problem devised by Mas Colell (Wilson [7] is concerned with three consumers and two goods. By using the theory of Section II.2 of Elken [2] we see that solving the equilibrium problem devised by Mas Colell is equivalent to solving (1) with

$$D = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ .5 & 1. & .25 & 1. \\ 1. & .5 & .20 & 1. \end{bmatrix}, b = \begin{bmatrix} -0.9 \\ -0.95 \\ -3.92 \\ 3.0 \\ 3.0 \end{bmatrix}$$

and

$$C = \begin{bmatrix} -0.9 & 0 & 0 \\ 0 & -0.95 & 0 \\ 0 & 0 & -3.92 \\ 1. & 1. & 1. \\ 1. & 1. & 1. \end{bmatrix}$$

Notice that the sum of the columns of C is b. This will always be the case. We are almost ready to type the data cards for BCA, but we have to add a row to D to represent the objective row and assign names to the rows and columns. Below is our choice of names.

		ACT1	ACT2	ACT3	MEXP
D =	UTIL1	Γ-1			1
	UTIL2		-1		down 1
	UTIL3			-1	
	GOOD1	.5	1.	.25	1.
	GOOD2	1.	.5	.20	1.
	OBJ				1.

We only enter the nonzero values to emphasize the fact that only the nonzero elements of D and b need be entered.

In the sample input below (Figure 1) we specify that we want

three consumers (IH = 3),

two goods (L = 2),

no production (IPROD = 0),

and a

C matrix input check (ICECHO = 1) .

Notice that we specify the Ith column of C by reading the Ith element from b and placing it in the corresponding position of $C(\cdot,I)$, and we read in the 4th and 5th elements from the card following

Sample Output

The output for the solution of the Mas-Colell problem by BCA is on the following pages. The current form of the program is rather verbose -- an option should be put in to print only the final solution if that is all that the user wants to see.

The first page of output shows the values of all parameters, gives the problem name and the output concerning the solution of the auxilliary linear program. Messages concerning time can be ignored in this printout because no timing routine was being called.

```
0.5
           MRS-COLELL
            &PARM1 IH=3,L=2,IPROD=0,ICECHC=1, &END
  1.
  2.
            &PARAM IOBJ=-6, &END
                            MRS-CCL
           NAME
  4.
           ROWS
  5.
             L UTIL1
             L UTIL2
L UTIL3
  6.
   8.
             L 60001
  9.
             L 60002
             N CBJ
 10.
 11.
           COLUMNS
 12.
                           UTILI
               RCT1
                                                         60001
                                                                        0.5
                                            -1.0
               RCT1
                           60002
                                             1.0
. 14.
                           UTIL2
               RCT2
                                                         60001
                                            -1.0
                                                                        1.0
 15.
               RCT2
                           60002
                                             0.5
               RCT3
 16.
                           UTIL3
                                            -1.0
                                                         G00D1
                                                                        0.25
               RCT3
                           60002
 16.5
                                             0.20
 17.
               MEXP
                                                                        1.0
                           60001
                                             1.0
                                                         60002
               MEXP
                           CBJ
                                             1.0
 19.
           RHS
 20.
21.
               ENDON
                           UTILI
                                            -0.9
                                                         UTIL2
                                                                       -0.95
                                            -3.92
                           UTIL3
               ENDON
 22.
               ENDOW
                           60001
                                             3.0
                                                         60002
                                                                        3.0
 23.
           ENDATA
 24.
              0
                        0
 25.
                   4
             - 1
                        5
 26.
               1.
                           1.
 27.
 28.
              0
                   2
                        0
 29.
                   4
                        5
 30.
               1.
                           1.
 31.
 32.
-3.
              0
                   3
                        0
                   4
                        5
             -1
               1.
  4.
                           1 .
```

Figure 1

The following pages give information concerning the behavior of the bilinear complementarity algorithm, and finally, the solution. As a measure of the work done by the algorithm, the number of calls to subroutine ENDPNT, the number of scalar function evaluations, and the number of IX × IH jacobian evaluations are printed. The solution is printed in a common format for linear programs:

JH(I)	The names of the primal variables which may be non-zero					
	in the equilibrium solution					

VALUE gives the value of these variables. The value of MEXP should always be near zero in an equilibrium.

ROW NAMES is self-explanatory

PI(I) gives the value of the dual multiplier associated with that row. Those PI's associated with commodity balance rows are the usual "equilibrium prices."

RHS gives the value of the original RHS vector (b).

STATE STAT	.99999999999999997D-05,ICNTRL= 3	F= -1.00000000 .NRMAX=	99999, ZTOLRP= 5, TFB, 999999, STOLRP= , 999999975E-05, Z1 , L= 1, I CHO= 350, NTMAX= 1S= 100	0.25000000 S 15	0 C C C C C C C C C C C C C C C C C C C
3 1 10000000000000000000000000000000000)00005E-03.ZSCAL	• • • • • • • • • • • • • • • • • • • •	NNE GO	
3 1 0.0000000000000000000000000000000000	0-0100000000000000000000000000000000000		1.NDUAL = FRQ= .10000008E-0	00	1.0000000000000000000000000000000000000
** ** ** ** ** ** ** ** ** ** ** ** **	0	100	0.2TC		162 163 163 163
	10.000000000000000000000000000000000000	C.17L.	2000. NEMAX= NEMAX= 0.1FCRSH= 1.1GBJ= C0016F-01. ZETA= .10000043E-11.KINB= EEND PROBLEM STATISTICS 4 STRUCTURAL COLUMNS 12 NON-ZERG ELEMENTS DENSITY = 0.5000 THE TIME LEFT IS NOW 1.000000 SECINVERT STATISTICS 14 NONZ IN BASIS 14 NONZ IN BASIS 14 STRUCTURAL COLUMNS IN EASIS 15 NONZ IN BASIS 16 STRUCTURAL COLUMNS IN EASIS 17 NONZ IN BASIS 18 NONZ IN BASIS 18 STRUCTURAL COLUMNS IN EASIS 19 NONZ IN BASIS 10 NONZ IN RASIS 10 NONZ IN BASIS	NINF (B) VALUE	VALUE C.9000000000000000000000000000000000000

STRUCTURAL COLUMNS OF MATRIX.

```
| COLUMNS | COLU
```

THE REAL PROPERTY.

```
P1(1)
0.63397455
0.86602845
0.23660254
0.23650290
0.26794910
                                                                                                                                                                                                                                                          .2264982
                                                                                                                                                                      -0.000000
                                                                                                                                                        -0.0001
                                                                                                                              1
                                                                                                                                           0
                                                                                                                             CCNSTRAINT TYPE:( 0 0.21268 0.67988 0.00011 0.007988 0.207016 0.207016 0.20470 0.20650 - 0.000000 0.000000 -
                                                                                                                                                                                                                                                         1.1547005
                                                                                              CURVE
                                                                                                                                                                                                                                                                                          S
                                                                                                                                                                                                                                                                                         ROW NAMES
UTILI
UTILI
UTILI
GOODI
GOODI
                                                                     ERC
                                                                                                                                                                                        THE BILINEAR CENSTRAINT HIT ZERO.
THERE ARE NOW . 3 BUDGET SURPLUSES AT ZERO
                                                                                                                                                                                                                   ECUILIBRIUM
                             0.0
                                                                                                                                                                                                                               37
                                                                                              THE
                                                                     AT
                                         SEC.
                                                                                      4TH ENDPNI CALL:
CURRENT LINEAR APPROXIMATION TO
A2(*U)+
-0.0263976
-0.0358613
0.2155483
0.9990081
                                                                                                                                                                                                                                                   SEC
                                                             THE BILLINEAR CENSTRAINT HIT ZERO.
HERE ARE NOW , 2 BUDGET SURFLUSES
                                                                                                                                                                                                                                                  1.5773504
0.866025
                                                                                                                                                                                                                                                                                         VALUE
1.57735041
1.15470049
4.22649817
6.5725041
-0.00000023
                UH(I)= 1 0 0 0.216948

T = 0.657648 0.216948

THE TIME LEFT IS NCW 1.000000

AFTER 4 NEWICN ITERATIONS.
                                                                                                                                                                                                                  CALLS, WE HAVE
                                                                                                                                                                                                                               SCALAR FUNCTION CALLS=
JACOBIAN EVALUATIONS=
          X NO"
                                                                                                                                                                 0-
                                                                                                                             STEPLENGTH= 0.079954

ITERATION (x= 0.63992

NORM(F(X))= 0.055999 F= 1 TERATION (x= 0.000169 F= 1 TERATION (x= 0.000169 F= 1 TERATION (x= 0.00000 F= 1 AFTER 2 NEWICK (TERATIONS)
                                                                                                                                                                            (J)
                                                                                                                                                                                                                                                 THE TIME LEFT IS NOW UTILITY LEVELS: 0.633975 MAS-COL
           *NO=
C INFC: N
                                                                                                                                                                                                                   ENDONT
          SUPERBASIC
NUH(I)= 3
WUH(I)= 1
                                                                                                                                                                                                                   4
                                    LAWBDA=
                                                                                                                                                                                                                               TUTALS:
                                                                                                                                                                                                                                                                                         ACTI
ACTI
ACTI
ACTI
UTILI
                                                                                                                                                                                                                  AFTER
                                                                                        THE
```

Sample Problem: Whisman

Next we give a sample input and output for a problem formulated by Al Whisman (Wilson [1976]). The problem involves four traders and three goods. Each trader is concerned with two to four activities. To describe the problem we give only the D, b, and C matrices of problem (1):

$$b^{T} = [-.48 \quad -.32 \quad -5.1 \quad -.95 \quad 5 \quad 11 \quad 9]$$
,

$$C = \begin{bmatrix} -.48 & & & & & \\ & -.32 & & & \\ & & -.51 & & \\ & & & -.95 & \\ 2 & 1 & 1 & 1 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$

Again, only the non-zero elements of the matrices are given.

The input follows. The first card, again, is just a header; it is not to be included in the input stream.

The sample output for the Whisman problem also follows, page 40.

```
WHISMAN
            &PARM1 IH=4.L=3, IPROD=0, ICECHC=1. &END
            &PARAM ICBJ=-8, &END
           NAME
                             WHISMAN
104.
           RONS
105.
             L UTIL1
106.
             L UTIL2
107.
             L UTIL3
108.
             L UTIL4
109.
             L 60001
110.
             L 60002
111.
             L 60003
112.
             N OBJ
113.
           COLUMNS
114.
               CIRCI
                            UTIL1
                                            -1.0
                                                          60001
                                                                         1.0
               CIRC1
CIRC2
115.
                            60002
                                             2.0
                                                          60003
                                                                         3.0
116.
                            UTIL1
                                            -1.0
                                                          600D1
                                                                         2.0
117.
               CIRC2
                                                                         2.0
                            60002
                                             2.0
                                                          60003
118.
               C2RC1
                                            -1.0
                                                                         4.0
                            UTIL2
                                                          60001
               CZRC1
119.
                            60002
                                             3.
                                                                         1.
                                                          80003
120.
               C2RC2
                           UTIL2
                                            -1.0
                                                         60001
                                                                         3.
121.
                                             2.
               C2RC2
                            60002
                                                                         2.
                                                         60003
               C3RC1
                            UTIL3
                                            -1.
                                                         60002
                                                                         1.0
                                            -1.
               C3RC2
                           UTIL3
                                                         600D1
                                                                         1.0
124.
               C3RC2
                            60002
                                             2.
125.
                                            -1.
               C3RC3
                           UTIL3
                                                         60002
                                                                         3.
126.
               C3RC3
                            60003
                                             4.
                                            -1.0
               C3RC4
                            UTIL3
                                                         60001
                                                                         3.
128.
               C3RC4
                           60003
                                             3.0
129.
               C4RC1
                           UTILY
                                            -1.0
                                                         60001
                                                                         1.
130.
               CHRC1
                           60002
                                             1.0
131.
                                            -1.
               04802
                           UTIL4
                                                         600D1
                                                                         2.
132.
                                            1.
                                                         60002
                                                                         1.
               MEXP
                           600D1
133.
               MEXP
                           60003
                                             1 .
                                                         CBJ
                                                                         1.
          RHS
134.
135.
                           UTIL1
               ENDOW
                                            -0.48
                                                         UTIL2
                                                                        -0.32
136.
                           UTIL3
                                            -5.1
               ENDOM
                                                         UTILY
                                                                        -0.95
                                             5.
137.
               ENDOW
                           60001
                                                         60002
                                                                        11.0
138.
                                             9.
               ENDOW
                           60003
139.
          ENDRIR
              0
                        0
141.
                   5
                        7
               2.
142.
                           2.
                                       1.
143.
144.
                   25
              0
                        0
145.
146.
                           3.
               1.
                                       3.
147.
148.
                   3
              0
149.
                   5
150.
               1 .
                           5.
                                       4.
151.
152.
              0
            -1.
                   5
154.
                           1.
155.
```

1H= NY WAX=	5,1FP	350.NTMAX= AS= TOLR1= .100	NFTA NELEM 55 22 7 226 7 226 9 337	
<i>:</i>	=dN IX 60666	99999,710LRP= .99999 .L= .L= AS= TOLR	-6.50000000 -2.50000000 -1.70000000 -1.7400000 -0.37500000	000000
500000000000000000 .ICECHO=	=MIdIN*666	0 • 17 PL 1 M=	00 00 00 00 00 00 00 00 00 00 00 00 00	0000000 -0.48000000 0000000 -0.32000000 0000000 -0.95000000 0000000 -0.950000000 0000000 -0.9500000000000000000000000000000000000
00000 ,STPRD= ,500000 WE= 100 100000034E-05,ZTCJST= .1	I.NDUAL =	30.INVFFQ= 3	VECIN VECOUT C3AC3 MEXP C3AC4 G00D2 MEXP G00D1 C2AC2 C2AC1 G00D3 C3AC4 G00D3 C3AC4	TENO
171	= 0.NDEGI=	B.ITCH= NB= .000000 SEC. N EASIS	EBJ VALUE 9.15959839 0.5899997 -0.58874998 -1.54874992 -2.60999966	ALUE 0.48000000 0.32000000 0.73000000 0.7112 0.71
0.STPMX= 0.KnUTR= 0.IH.ICFCFC = .1000.0C005E	2000. A000.IALG=	TITORDEC 43E-11 WHISMAN STATISTICS DWS NASCHER ELEMENT STATISTICS	UNT STATUS NINF	VAP CO CO CO CO CO CO CO CO CO CO CO CO CO
EEND LECHO. EPARAM 270L ZE=	NEMAX=	2000016E-0 2END 2END 8 RD 11 SI 11 SI 12 SI 13 SI 14 E TIM 1 NVE NI S 2 VEC 6 VEC 6 VEC 6 VEC 6 VEC 6 VEC 6 VEC 6 VEC 7 SI SI 8	TTCOUNT 2 2 3 3 4 4 5 5 5 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6	014(1) 02402 03401 04401 06003 06003 061

STRUCTURAL COLUMNS OF MATRIX.

```
THE 2TH ENDPAIT CALL:
THE CURRENT LINEAR APPROXIMATION TO THE CURVE IS:

A2(*U) +

A3

1.000)000

STCPLENGTH=

ITERATION

NORM(F(X)) = 0.0

AFER 0 NEWTON ITERATIONS.

THE 18TH DUAL VARIABLE WENT TO ZERO.
                                                                                                                                                                                                                                            0.113333 CENSTRAINT TYPE:(
                                                                                      THE THE ENDERT CALL:

THE CURRENT LINEAR APPEDXIMATION IC THE CURVE

A2(*U)+

1.000.000

STEPLENGIN 0.740000 CCNSTRAINT TYPE:(
ITERATION 0.740000 CCNSTRAINT TYPE:(
NORM(F(X))= 0.0740000 F= 0.074000

AFTER 3 NEWITON ITERATIONS.

THE 1STH PRIMAL VAPIABLE WENT IC ZERO.
VECIN: UTIL1
                                                                                                                                                                                                                                 THE BILINEAR CONSTRAINT HIT ZERG.
THERE ARE NOW . 1 BUDGET SURPLUSES AT ZERG.
                                                                                                                                                                                                                                                                                           SUPERBASIC INFC: KNU= 1 KMU=

NUH(I)= 1 0 0 0

MUH(I)= 0 0 0 0

LAMBDA= 1.5CCC00

THE TIME LEFT IS NGW 1.0000CC

AFTER 0 NEWICN [TERATIONS.
                           1 FUG GH
                           41
```

CCNSTRAINT TYPE: (

400 NO - N 40 00 00 - N 40 00 00 00 00 00 00 00 00 00

1.000000

0.0 0.500000 SEC.

0

KMC

6

0.

```
000000000
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              CCNSTRAINT TYFE:( -1, 11)

C.46757 0.37856

0.036484 0.031383 0.000

0.46000 0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              NUMBER
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                THE CURRENT LINEAR APPROXIMATION TO THE CURVE IS:

A2(*U)+
A2(*U)+
A2(*U)+
A3(*U)+
A3(
         CURRENT LINEAR APPROXIMATION IC THE CURVE IS:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              THE CURRENT LINEAR APPROXIMATION TO THE CURVE IS:

A2(*U)+
A2(*U)+
A3
A2(*U)+
A3
A3437402
0.2934162
0.3831918
0.02934162
0.8351918
0.0463310 CCNSTRAINT TYFF:(-1)
TERATION 0 x= 1.50010 0.46757 0.37856
NORM(F(X))= 0.047529 F= 0.036484 0.331383
NORM(F(X))= 0.0 0.0 F= 0.0
                                                                                                                                                                                            CCNSTRAINT TYPE: (
0.11333
C.0 0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        THE BILINEAR CCNSTRAINT HIT ZFRC.
THERE ARE NOW . 2 EUDGET SURPLUSES AT ZERG.
                                                                                                                                                         STEPLENGTH= 0.113333 CCNSTRAINT ITERATION 0.113333 CCNSTRAINT NORW(F(X))= 0.0 55333 0.11333 0.00 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.11333 0.113
                                                       0.113333 CCNST
0.113333 CCNST
0.85333
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     S SOC
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  INFG:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  0-
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  SUPERBASIC
NUH(I)= 0
MUH(I)= 1
```

```
966666660
                                                                                                                                                                                 0.00000.0
                                                                                                                                                                                                                                                                   -1.00000000
                                                                                                                                                                                                                                        1.7000000
                                                                                                                                                                 0005000
                                                                                         00000000
 00000001
                                                                              6
                                                                              0
                                                                                                                                     18:
                                          THE 7TH ENDPNI CALL:
THE CURRENT LINEAR APPROXIMATION IG THE CURVE IS
A2(*U)+
0.40
0.40
1.000000
0.3366666
0.853334
0.853334
0.853333
NORM(F(X))= 0.000000
0.85333
AFTER 0 NEWICN ITERATIONS.
                                                                                                                                                                     CCNSTRAINT TYPE: (
C.5C000 0.66333
-0.000000 0.000000
                                                                                                                                                                                                                                        0.8000000
                                                                                                                                     CURVE
                                                                                                                                                                                                                                                                   UTIL2
UTIL2
UTIL3
UTIL4
GCOD1
GCOD2
GCOD3
THE TIME LEFT IS NOW 1.366667

THE TIME LEFT IS NOW 1.30000 SEC.

AFTER 1 NEWTON ITERATIONS.

THE 18TH DLAL VECOLT: UTILI
                                                                                                              ZERC
                                                                                                                                                                                                       ECUILIBRIUM
                                                                                                                                                                                                                  20
                                                                                                                                     THE
                                                                                                               AT
                                                                                                                              THE BIH ENDENT CALL:

THE CURRENT LINEAR APPROXIMATION 10 TH

A2(*U) + 0.4800000
0.0 0.50000
0.0 0.853334
0.0 0.8533334
STEPLENGTH= 0.000000 F= -0.000000
AFTER 0 NEWTON ITERATIONS.
                                                                                                                                                                                                                                  1.3333334
2.503030
                                                                                                         THE BILINEAR CCNSTRAINT HIT ZERO.
THERE ARE NOW . 3 BUDGET SURPLUSES
                                                                                                                                                                                                                                                                   VALUE
1.3323337
(.80000001
6.00000001
0.000000000
3.399986
0.26666697
                                                                                                                                                                                                                 SCALAP FUNCTION CALLS=
JACOPIAN EVALUATIONS=
                                                                                                                                                                                                       ENDPNT CALLS. WE HAVE
                                                                                                                                                                                                                                 UTILITY LEVELS:
PRICES: 1.50000
                                                                                                                                                                                                                  TOTALS:
                                                                                                                                                                                                                                                                    C1AC1
C2AC2
C3AC1
C4AC1
MEXP
600003
C4AC2
                                                                                                                                                                                                       AFTER
                                        THE
43
```

II.5. Source Listing for BCA

The following pages are a listing of the program BCA as it is written in FORTRAN IV. The subroutines which make up LPM1 (Tomlin [1975]) are omitted.

```
IMPLICIT REAL*8 (A-H.O-Z)
INTECER PD.PO1.PD2.SS.FR.ZFLAG.RS.P.DD.DD1.BL1.BL2
INTECER*2 JH(350).DIGMA(952).KINBAS(1302).IDBAS(1302).
INTEGER*2 ISTYPE.LA.LE.IA.IE.PUN.LC(20).IC(300)
DOUBLE PRECISION F(8000)
    1 .
    4 .
     5 .
    6.
                                                                                           A(4000) . C(80)) . CMIN . CEND . ERMAX . SUMINE
                                                                  REAL
    8 .
                                    C
                                                            COMMEN DSUN.DPFOD.CY.DE.DP.P(350).X(350).Y(350).YTEMP(350).

1A.E.CMIN.CCND.FPMAX.SUNINF.ICNAM(13)2.2).NAME(20).

2NTEMP(21).KINP.ITIM.JTIM.ITINV.JTINV.MSTAT.[03J.ROWP.IVIN.IVOUT.

3ITCNT.INVFFQ.ITRLIM.IFFEZ.JCOLP.NRGW.NCOL.NELEM.NETA.NLELFM.NLETA.

4NGELEM.NINF.NUELEM.NUETA.NNEGDJ.NL[NES.ISTYPE(350).

SLA(1302).LE(2002).PUN(8).
     9.
 10.
 11.
12.
 14.
 15.
                                                             6 IPUNC . NDEC I . NDUAL . NIPIW . IFEAS . IFCR SH
                                                          COMMON IICH, ITCHA, IFPIWT, IFNES, KOUTS
COMMON IA (4000). IF (8000)
COMMONZED (STZBET (10), BF2(10), E1(10), E2(10), C.IC.LC
COMMONZED (STZBET (10), BF2(10), E1(10), E2(10), E2(10), C.IC.LC
COMMONZED (STZZDI (10.10), D2(9.10), E1(9.10), E2(9.10)
COMMONZENCONSZGI (350.10), G2(400.10), EA(350), B8(400)
COMMONZENCONSZGI (350.10), G2(400.10), EA(350), B8(400)
COMMONZENCONZZ JH. DIGMA.K INEAS.IDEAS
CO4MONZENCONZZ JH. DIGMA.K INEAS.IDEAS
CO4MONZENCONZ JH. DIGMA.K INEAS.IDEAS
CO4MONZENCONZ JH. DIGMA.K INEAS.IDEA
CO4
 15.5
                                     (
16.
 18.
 19.
20.
 21 .
 22 .
 23.
24 .
20 .
 27.
28.
29.
30.1
                                                   ACA: A CODE WHICH INFLEMENTS THE BILINEAR COMPLEMENTARITY ALGORITHM FOR SOLVING ECONOMIC FOUILIERIUM PROBLEMS.
 30.2
 30.3
30.4
                                     C
                                                                                                                THEMAS Q. ELKEN
SYSTEMS OPTIMIZATION LABORATORY
                                                                 AUTHOR:
 30 .€
                                                                                                                OPERATIONS RESEARCH DEPARTMENT STANFORD UNIVERSITY
  10.7
 30.8
30.91
                                                   FOR A DISCRIPTION OF THE ALCORITHM AND DOCUMENTATION OF THIS PROGRAM SEE SOL TECHNICAL REPORTS 77-26 AND 77- .
 10.92
 30.93
                                                                 10LBD= 10-04
TOLCV= 10-05
 31 .
 33 .
                                                                  TOLFZ=
                                                                                                10-10
                                                                  STPMX= 10.0
 34 .
                                                                  STPRD= 0.5
ITLIME= 100
 35 .
36.
                                                                  ICNTRL=
                                                                  IECHC=
 34.
                                                                  KJUTH=
                                                                 ICECHC = KFUN= 0
40 .
41 .
42 .
                                                      IPPOD= 1
READ (5.PARNI)
WRITE (6.PARNI)
NRITE (6.9) IECHC.IH.ICECHO
FORMAT(IX.* IFCHC.IH.ICECHO
 43.
44 .
45.
46.
                                                                                                                            IFCHO. IH, ICECHO . 314)
```

```
IF (TCNIRL) 10.40.4)
10 CALL INPUT(IACT)
1F (1ACT) 30. 20. 20
20 IF (IECHO.EQ.X) GO TO 29
  48.
  50.
                         MUB= NRCW
WRITE (6.21)
21 FOPMAT (////.* STRUCTURAL COLUMNS OF MATRIX.*)
  53.
  54 .
  55.
                          22 LB= MUE+1
  56.
                                MUSE LET 14
                               K= 0

DO 25 J= LB+MUB

IF (J+GT+NCOL) GC TO 26

K= K+ 1

CALL UNPACK(J)

DO 24 I= 1+NEG%

C1(I+K)= V(I)
  58 .
  59.
  63.
  61.
  62.
  63 .
  64 .
                         24 CONTINUE

25 CONTINUE

26 WRITE (c.27) L8.NUB

27 FORMAT (////.' COLUMNS '.14." THRUDGH '.14)

00 23 I=1.NROW

MRITE (c.28) (GI(I.J).J=1.K)

28 FORMAT (IX.15F8.3)
  65.
  67.
  68 .
  59.
  71.
                        23 CONTINUE
1F (MUH .LT. NCOL) GO TO 22
29 CALL NEFMAL
CALL UNKAVL())
GG TO 10
30 STOP
  74 .
  75 · 76 · 77 ·
                 C
                         43 CALL INPUT(IACT)
CALL DEHUG(I)
CALL NORMAL
CALL UNRAVL(C)
  78 .
  79.
  8).
  81 ·
82 ·
83 ·
                  (
                               CALL DEBUG(1)
CALL SHIFTR(3,4)
KEND= 0
  84 .
  85.
                               KEND= 0
KENDSV= 0
IHL= I++ L
M= NRCW
N= NCCL - NROW
NM= N+ M
L1= M- L
  86.
  88.
  89 .
  90 .
  91.
                         MM1 = M- 1

ITSINV= 1

IF (IPROD.60.0) GO TO 43

READ (5.1030) (SER(I).I=1.IE)

43 ICOU=0
  92 .
  93.
  94.
 96.
                             98.
  99 .
100.
101 .
102.
103.
107.
```

```
108.
109.
110.
111.
                                                         C(ICOU) = E(LL)
                                               GC TC 45

DU 48 I= LL, KK

ICOU= ICCU+ 1

IC(ICOU)= E(I)
                              47
113.
                                               CONTINUE
GO TO 45
                              48
114.
                                            GO 10 45

IF (KK.NE.)) GO TO 44

ICCU= ICOU+ 1

IC(ICOU)= LL

REAC (5,10 30) C(ICCU)

GO 10 45

ISAV= ICCU+ 1

DC 46 != LL.KK

ICCU= ICOU+ 1

IC(ICOU)= I

CONTINUE
115.
                              42
118.
119.
120 .
121 .
                              44
122.
123.
124.
125.
126.
127.
128.
                              46//
                                            CONTINUE

PFAC(5.133)) (C(J).J=[SAV.ICOU)

GC T( 45

IF (KK.NE.)) GO TO 53

ICCU= ICCU+ 1

IC(ICCU)= LL

C(ICCU)= SHR(K)*P(LL)

GO TO 45

DO 45 I= LL, KK

ICCU= ICCU+ 1

C(ICCU)= SHR(K)*P(I)

IC(ICCU)= ICCU+ 1

CONTINUE
                                                CONTINUE
                              50
129.
130.
131.
133.
                              53
135.
136.
                              55
                                             CONTINUE
                                            GC 10 45
LC(KP1)= 1C0U+1
138.
139.
                             140.
142.
143.
                                                                                       STRUCTURAL COLUMNS OF MATRIX. 1)
144 .
145 .
                                      MUB= LB+ IH - 1
140.
                                    K= 0

DO 65 J= LP.MUB

IF (J.5T.NCOL) GC TO 66

K= P+ 1

CALL UPAKC(J)

DO 64 I= 1.NROW

G1(1.K)= Y(1)
147.
150.
151 .
 153.
154 ·
155 ·
156 ·
                                                CENTINUE
                             64 CENTINUE

65 CONTINUE

66 WRITE (6.67) LB.MUB

67 FURMAT (////.' COLUMNS '.14.' THRUOGH '.14)

DG 74 I= 1.NROW

WRITE (6.73) (G1(I.J).J=1.IH)

73 FURMAT (1X.10F10.4)
157 .
158.
159.
                              73 FURMAT (1X,10F1
74 CONTINUE
69 DO 70 I= 1.M
IR= JH(1)
XX(IR)= X(1)
PI(1)= YIEMP(1)
HA(1)= X(1)
70 CONTINUE
161 .
 162.
163.
164.
                              70 CONTINUE
157.
```

```
MP1= M+ 1
D0 100 J= MP1,NM
1F (KINHAS(J).NE. 0) GC 10 90
DSUM= 0.
LL= LA(J)
KK= LA(J+1) - 1
00 00 I= LL.KK
IR= IA(I)
DE= A(I)
DPPGC= DE*YTEMP(IR)
DSUM= DSUM + DPR(D
CONTINUE
165 .
169.
171 .
173.
174.
176 .
177.
 178.
                           80 CONTINUE
PT(J) = DSUN
GU TO 100
90 PI(J) = 1.
179.
 181 .
182 .
                          100 CONTINU
183.
                         100 CONTINO

K= 0

DO 113 I= 1.NM

IF (KINMAS(I) .EQ. 0) GO TO 105

IDBAS(I)= 0

GO TO 110

1)5 K= K+ 1

DIGMA(K)= I
134 .
185.
186 .
187.
188.
189.
190.
                                    88(K)= PI(I)
IDBAS(I)= K
191.
192.
                         IDBAS(1) = K
XX(1) = J.)

110 CONTINUE
THIS IS THE BILINEAR PHASE OF THE ALGORITHM. VARIABLES XX(1)....
XX(M) ARE THE SLACKS. AND PI(1)....PI(M) ARE THE USUAL PI'S.
XX(M+1)....XX(M+N) ARE THE X*S. PI(M+1)....PI(M+N) ARE THE
DUAL SLACKS. MORE INITIALIZATIONS.
194 .
195 .
196.
                     C
198.
199.
200.
                                    00 120 I=1.IH
                                   MUH(I) = 0
MUH(I) = 0
201.
202.
                                   NU(1)= )
MU(1)= 0
204 .
235.
                          12) CONTINU
207.
                                   KNU= 0
                                    KMU= 0
                                   00=1
209.
                          420 90= 1
                                               IF (KINEAS(CC).GT.C) GO TO 425
CALL SUPERB(1.1.DO.O.O)
GC TO 430
CALL SUPERB(1.-1.CD.C.C)
210.
211.
212.
                         425
                                               CALL
214.
                                            PD= -1
                          430 JS= DD
215.
216.
                                   CALL BLCONS
                                   ZFLAG= 0
GD TC 460
219.
                         GO TC 460
450 NTEMP(1) = NTEMP(1) + JTINV
CALL INVERT
CALL RECALC
ITSINV= 0
450 KEND= KEND+ 1
ITSINV= ITSINV + 1
IF (KEND - GT - ITLIME) STCP
WRITE (6.222) KEND
222 FORMAT (/.*
219.
220.
221.
121.
224.
120 .
```

```
1./.' THE '.14, TH ENDENT CALL:')

CALL ENDENT(JS, POI, IS, NET)

IF ( (KEND/3)*3 .NE. KEND ) GO TO 468

CALL DEFUG(4)

CALL DEBUG(1)
423.
229 .
230 .
231.
                             CALL DEBUG(1)

468 WRITE (5.470) NET

470 FORMAT (1X.' AFTER '.13.' NEWTON ITERATIONS,')

IF (PO1.8G.1 .AND. IS.EG.NN) GO TO 920

IF (PD1) 300.306.310

300 WRITE (5.205) 15

305 FORMAT (1X.' THE '.13.'TH CUAL VARIABLE WENT TO ZERO.')

GO TO 450

306 WRITE (6.307) DO

307 FURMAT (/.1X.' THE BILINEAR CONSTRAINT HIT ZERO.'./

1.' THERE ARE NOW .'.13.' BUDGET SURPLUSES AT ZERO.')

IF (DD .EG. IH) GC TO 920

DD= DD+ 1

GO TO 42)

310 WRITE (6.715) IS

315 FURMAT (1X.' THE '.13.'TH PRIMAL VARIABLE WENT TO ZERO.')

480 IF (PD1.8G.-1) GC TO 520

IF (15.8G. DD) GD TO 490
233.
 234 .
235.
236.
238.
 239.
240 .
241 .
242 .
243.
244.
£46.
247 .
 248.
249 .
250.
                                   A PRIMAL VARIABLE WENT TO ZERO. THAT VARIABLE MUST GO TO NUMBASIC. AND THE INCOMING VARIABLE IS DETERMINED BY FINDING THE LARGEST ELEMENT IN THE AFFRORRIATE ROW OF GI.
251 .
252 .
253 .
255.
                                         PO= -1
                             PO= -1

483 IROWP= KINFAS(IS)

CALL FINDF(1.IS.J(OLF)

INNAM1= ICNAM(JCOLP.1)

INNAM2= ICNAM(JCOLP.2)

IUNAM1= ICNAM(IS.1)

IONAM2= ICNAM(IS.2)

WRITE(6.485) INNAM1.INNAM2.IONAM1.IONAM2

485 FORMAI(IX.' VECIN: '.2A4.' VECOUT: '.2A4)

CALL PIVCT(JCOLP.IS)

CALL UNDACK(JCOLE)
256.
 257.
258 .
259.
260 .
 261.
 262.
263.
205 .
                                         CALL UNPACK (JCOLF)
                                        CALL FTFAN(1)
CALL WRETA
CALL BSCNC(JCOLP.IS)
CALL SUPERB(0.-1.JCOLF.1.JCOLP)
CALL BLCCNS
266.
268.
269.
 270 .
271 ·
272 ·
273 ·
                                         JS= IS
IF (ITSINV .GE. INVERC) GC TO 450
GU TC 460
274 .
                              490 10S= DD
                                         DD= DD -1
 275.
276 .
                                         J5= CO
I (DD.FG.0) STOF
178.
                                         PD=
                                        IF (KINBAS(IDS) .NE. 0) 60 TO 530 CALL SUPFER(2.0.0.1.10°) GO TO 400
 279.
280 · 281 · 282 ·
283 .
                              500 CALL SUPPRE(2.0.0.-1.105)
                                        GU TO WHY
 284 .
                        CC
285.
                                  A DUAL VARIABLE WENT TO ZERO. THAT VARIABLE MUST ENTER THE BASIS. AND THE LEAVING VARIABLE MUSY BE DETERMINED BY FINDING
286.
237 .
```

```
.885
                                    THE LARGEST PIVOT FLEMENT IN THE IDBAS(IS)-ROW OF G2.
                            520 PD= 1
IF (IS.EQ.OD) GO TO 530

523 JCDLP= IS
CALL FINDE(-1.IS.P)
IROWP= KINFAS(P)
INNAM1= ICNAM(IS.1)
INNAM2= ICNAM(IS.2)
IUNAM1= ICNAM(P.1)
IONAM2= ICNAM(P.2)
WPITE (6.485) INNAM1.INNAM2.IONAM1.ICNAM2
CALL PIVOT(JCOLP.P)
CALL UNPACK(JCOLP)
CALL FIRAN(1)
CALL #RETA
CALL BSCN((JCOLP.P)
CALL SUPERE(0.1.F.-1.F)
CALL SUPERE(0.1.F.-1.F)
CALL SUPERE(0.1.F.-1.F)
CALL BSCN(JSCN)
JS= IS
289.
290.
292.
293 .
194 .
295.
297 .
298.
299.
300.
 303.
304 .
305.
330.
                             JS= IS

IF (ITSINV.GE.INVERO) GO TO 450

GD TO 460

530 PD= )
.107.
 3)8 ·
310.
                                         IDS = DC
511 .
312.
                                         DD= DD-
                                         JS= DD

IF (DD.EG.0) STOF

IF (ICPAS(IDS) .NE. 0) GC'TC 540

CALL SUPERP(2:0:0:-1:IDS)

GO TO 460
 313.
314.
315.
316.
317.
318.
319.
520.
                             540 CALL SUPERB(2,0.0.1.10$)
321.
                            920 WRITE (6,930) KEND
930 FORMAT(//,* AFTER*.14.F ENDENT CALLS, WE HAVE EQUILIBRIUM.*)

KJAC= KJAC/ (IH*IH)

WRITE (5,320) KFCN.KJAC

320 FORMAT(/,* TUTALS: SCALAR FUNCTION CALLS=*.IB./.IIX.*JACOBIAN FVA

1LLATIONS=*.18./)

CALL DEBUC(1)

DO 935 I= 1.IH

935 VS(I)= -8(I)+ XX(I)

WRITE (6,940) (VS(I).I=1.IH)

WRITE (6,945) (PI(I).I=1.IH)

940 FORMAT (IX.* CTILITY LEVELS: *.6F15.7)

945 FORMAT (IX.* PRICES: *.9F12.6)

DO 950 I= 1.NRCW

IV= JH(I)

X(I)= XX(IV)

Y(I)= PI(I)
322.
                        C.
 124 .
325.
 326.
 327.
329.
330 .
331 .
332.
334 .
 135 .
 336.
338.
                                                 Y(1)= P1(1)
339 .
                              950 CONTINUE
 340 .
                                        CALL UNRAVE(0)
 341 .
342 .
                           1010 FORMAT (1814)
1030 FORMAT (10F7.4)
343 .
.544 .
                                         END
 $45 .
                                         SUBROUTINE BLCONS
346.
 347 .
                        C.
```

```
WE ARE GOING TO CALCULATE THE DD ROWS OF COLFFICIENTS FOR THE BILINEAR EQUATIONS AND THE BILINEAR INEQUALITY. THE BASIC MATHEMATICAL STRUCTURE IS THE FOLLOWING:
348 .
.149 .
350.
 351 .
                                          C
                                                                                BF1 + D1*P1(MU) - DIAG(PI(MU))*(F1*X(NU) + E1) = BF2 + D2*P1(MU) - DIAG(X(NU))*(F2*PI(MU) + E2) =
 152.
 353 .
 354 .
                                          C
                                                                       IMPLICIT REAL *8 (A-H.O-Z)
355 .
                                                                      INTEGER #8 .HO 1. PE2.55.F6.7FLAG.RS.P.CD.DD1.BL1.EL2
INTEGER #2 JH(350).DIGMA(922).KINBAS(1332).(D6AS(1332)
INTEGER #2 ISTYPE.LA.LE.LA.TE.PUN.LC(20).IC(8C3)
DOUBLE PRECISION E(800)
REAL A(4000).C(800).CMIN.COND.ERMAX.SUMINF
 356.
  357 .
 358 .
 359 .
360 .
 .161 .
                                                                  CCMMCN DSUN, DPROD.DY, DE.DP, P(350), X(350), Y(350), YTEMP(350),
1A.E.CMIN.CCND.ERMAX.SUMINF.ICNAM(1302.2).NAMF(20).
2NTEMP(20).KINF, ITIN.JTIM.ITINV.JTINV.MSTAF.108J.IROWP.IVIN.IVOUT.
3ITCNT.INVERO.ITRLIM.IFFEZ.JCOLP.NRCW.NCOL.NELEM.NETA.NLELEM.NLETA,
 362 .
 503.
 364 .
 34,50
                                                                 366.
 557 .
 368 .
309.
  370 .
371 ·
372 ·
373 ·
.574 .
                                                                      COMMONZENCE THEN HELD STATE THE STATE OF THE
 3/5.
  $76 .
 577 .
378 .
379.
                                                                       K 1 =
                                                                       K2= 0
 380 .
                                                                      HL1= 0
HL2= 0
INO= MUH(CC)
INP= NUH(CD)
  381 .
  182 .
 383 .
  384 .
                                                                      1F (KMU, EG. 3) GO TO 103
00 80 K= 1.KMU
  385.
 386.
                                                        00 80 K= 1*KMU
K1= NU(K)
IF (K1*ST*IH) GO TO 80
IDD= KINHAS(K1)
BL1= BL1+ 1
IF (KNU*EC*0) GO TO 45
00 40 I= 1*KNU
40 F1(K*I)= G1(IDD*I)
45 E1(K)= B4(IDD)
CALL UPAKC(K1)
 387 .
 388.
 359.
 390 .
 .591 .
  192.
  393.
  394 .
                                                                       CALL UPAKC(K1)
DO 60 I= 1.KMU
DSUM = 0.0
  395.
 390 .
                                                                       DSUM
                                                                                                           0.0
 397 .
                                                                      IF (MU(I) *LE* M) DSUM= Y(M

DC 50 J= 1*M

IDJ= ICEAS(J)

IF (IDJ *EQ* C) GO TO 50

CSUM= CSUM+ Y(J)*G2(IDJ*I)
                                                                                                                                                                     DSUM= Y(MU(I))
  398.
 399.
 400 .
 401 .
 402.
                                                          SO CONTINUE
 403.
                                                         DI(K.I) = DSUM
60 CONTINUE
 4)4.
405.
                                                                        SF1(K) = 0.
 +35 .
 407.
                                                                                    70 J=
```

```
428.
                                   IF(IDBAS(J).EQ.O) GC TE 70
                                                                              Y(J)*RE(ICBAS(J))
409.
                           BF1(K) = BF1(K)+
70 CONTINUE
410.
                                   IF (KI.NE.DD) GO TO 80
INEG= 1
411.
413.
                                  BL 1 = BL 1 - 1
414.
                           80 CONTINUE
415.
                           NOW THE SECOND TYPE OF EQUATION IS CALCULATED BECAUSE PI(K) IS EASIC.
416.
417.
 +13.
                         1)0 IF (KNU.EC.0) PETURN
00 145 K= 1.KNU
K2= NU(K)
419.
420.
421 .
                                  IF(K2.CT.1F) GC TO 145
10D= IDDAS(K2)
BL2= BL2+ 1
422.
423.
                         BL2= BL2+ 1
IF (KMU.EG.J) GO TO 123
DO 120 I= 1.KMU
120 F2(K.I)= G2(IDD.I)
123 E2(K)= HB(IDD)
CALL UPAKC(K2)
IF (KMU.EG. 0) GC TO 132
DO 13) I= 1.KMU
D2(K.I)= C.C
IF (MU(I) .LF. M) D2(B
425.
426 .
427.
423.
429 .
                        D2(K.I) = C.C

IF (MU(I) .LE. M) D2(K.I) = Y(MU(I))

D0 125 J = 1.M

IF( 1DBAS(J) .EO.C) G0 T0 125

D2(K.I) = D2(K.I) + Y(J) *C2(ICBAS(J).I)

125 CONTINUE

130 CONTINUE

132 BF2(K) = 0.
430 .
431.
433.
435.
430.
+34.
                                  BF2(K)= 0.

DC 140 J= 1.M

AF (1DBAS(J).E0.0) GC TD 140

AF (1DBAS(J).E0.0) GC TD 140
439 .
441 .
                         ### HF2(K) + Y(J) ### (IDEAS(J))
140 CONTINUE
442.
                         IF (K2. NE. DD) GC TC 145
INEG= 2
BL2= BL2- 1
145 CONTINUE
445 .
447.
444.
                                  RETURN
                                  END
450.
                                   SUBROUTINE FINDP (PD1.15.P)
451 .
                           THIS SUBPOLITINE CHOOSES THE VARIABLE TO SECOME IMPLICITLY BASIC AS THE CNE WITH THE PIVOT ELEMENT LARGEST IN ABSOLUTE VALUE
452 .
454 .
4500
                                   IMPLICIT REAL #8 (A-H.O-Z)
456 .
                                  IMPLICIT REAL #8 (A-H,G-Z)
INTEGER PD.*PDI.*PU2.*SS.FF.ZFLAG.*RS.*F.CD.*DDI.*BLI.*BL2
INTEGER*2 JH(350).*DIGMA(952).*KINBAS(1302).*IORAS(1302)
COMMENZINCENSZGI(350.10).*GZ(400.10).*EA(350).*BB(400)
COMMENZINCXIZ JH.*CIGMA.*KINGAS.*IDBAS
COMMENZINZZZ JH.*CIGMA.*KINGAS.*IDBAS
COMMENZINZZZ JH.*N.*M.*DD.*KMU.*KNU.*BLI.*BLZ.*MP1.*NM.*INO.*INP
COMMENZINZZ TOLEZ.*TCLED.*TCLCV.*STEMX.*STPRD
458 .
457.
450 .
401 .
452 .
463.
                                  P= 0

IF (PD1.EG.-1) GC FC 3C

IDD= KINDAS(IS)

DO 20 I= 1.KNU
+64 .
465.
4000
```

```
IF (P.NE.C) GU TE 10
BIG= CARS(G1(IDC.I))
P= NU(I)
468.
469 .
470 .
                      GD TO 20
10 CDMP= DABS(GI(IDE:I))
IF (CDMP:LE:BIG) CO TO 20
BIG= COMP
471 .
472.
474 .
475 .
                            P= NU(1)
476.
                      20 CONTINUE
                            IF (BIG.LT.TOLFZ) P= 0
RETURN
478.
                      RETURN

3) IDD= ICMAS(IS)
DO 40 J= 1.KNU
IF (P.NF.0) GO IC 35
BIG= DAMS(GP(IDD.J))
P= MU(J)

35 COMP= DAMS(G2(IDC.J))
479 .
480 .
481 .
482 .
483 .
484 .
                            IF (CDMP.LF.BIG) GO TO 4)
BIG= CEMF
P= MU(J)
485 .
486.
                      40 CONTINUE
488.
489.
                             (F (BIG.LT.TOLFZ) P= 0
                            RETURN
490 .
491 .
                            END
492.
                            SUBROUTINE PIVOT (SS.FR)
493.
494.
                      THIS EQUITINE PEFFORMS A PIVOT ON THE PRIMAL SUPERBASIC COLUMNS IF PO.EQ.1. DUAL IF PO.EQ.-1. THE PIVOT BRINGS COLUMN SS INTO THE PASIS AND COLUMN RR DUT OF THE
495 .
                 C
496 .
497.
                      BASIS.
498.
                           IMPLICIT REAL*8 (A-H,O-Z)
INTEGER PO.PDI.PC2.5S.RR.ZFLAG.RS.P.CD.DDI.RLI.BL2
INTEGER*2 JH(350).DIGMA(952).KINHAS(1302).IDPAS(1302)
INTEGER*2 ISTYPE.LA.LE.IA.IF.PUN.LC(20).IC(800)
DOUBLE PRECISION E(8000)
REAL A(4000).C(800).CMIN.COND.EPMAX.SUMINE
499 .
500.
501 .
50% ·
503.
504 .
                         506.
                 C
507.
508.
509.
510 .
011.
512 .
513.
514 .
515.
510 .
518 ·
519.
520 .
521 .
                        IF (SS .GT. IH) GO TO 5
IF (NUH(SS).NE.O) GC TO 20
5 CALL UNPACK(SS)
CALL FIRAN(1)
523.
524 .
525.
                            ZB (RB) = -1./Y(RB)
DD 10 I= 1.M
527 .
```

```
IF (I.NF.RE) ZE(I)= Y(I)*ZE(RB)
528.
                            10 CONTINUE

GO TO 30

23 JS= NUF(SS)
529 .
530 .
                           23 JS= NUF(SS)
    ZB(RB) = 1./G1(RB.JS)
    DO 25 I= 1.M
    IF (1.NF.RB)    ZB(1) = -G1(I.JS)*Z3(RB)
25 CONTINUF
36 IF (KNU.EC.O) GO TO 55
    DU 50 J= 1.KNU
    IF (NU(J).EG.SS) GC TO 50
    V= G1(RH.J)
    G1(RB.J) = 0.
    DO 40 I= 1.M
40 G1(I.J)= C1(I.J)+ V*ZE(I)
    G1(PB.J)= -G1(PB.J)
50 CONTINUE
50 V= BA(RB)
531 .
532 .
533.
534 .
535.
536.
537.
538.
539.
540 .
543.
544 .
545.
                                   V= BA(RH)
                            BA(RB) = 0.

00 60 I= 1.M

60 BA(I) = BA(I) + V*ZB(I)

BA(RB) = -EA(RB)
546 .
047.
348 .
549.
 50.
                            WE ARS NOW PIVOTING ON THE DUAL SYSTEM. IF TH
IS NOT AN IMPLICIT EASIC-TYPE PIVOT, WE CALL SUPERB
TO CALCULATE THE PIVOT COLUMN.
 051.
                     C
552 .
753.
 754 .
                                  IFL= 0
RH= IDBAS(SS)
IF (RR .GT. IH) CG TC 70
IF (MUF(RR).NE.3) GO TC 90
555.
556 .
557 .
333 ·
                            70 IFL= 1
CALL SUPERE(1,-1.RR.0.C)
554.
                           CALL SUPERE(1.-1.RR.O.C)

JS= KMU
GU TO 82

BO JS= MUP(RR)

BZ ZB(RB)= 1./G2(RB.JS)

DO E5 I= 1.N

IF (I.NE.RB) ZB(I)= -G2(I.JS)*ZB(PB)

55 CDNTINUE
561 .
562 .
563.
764.
565.
567.
                                   IF (KMU.EC.J).GO TO 115
DO 110 J= 1.KMU
IF (MU(J).EQ.RR) GO TO 113
368 .
569.
570.
                         IF (MU(J).EQ.RR) GQ TQ 11

V= G2(RB.J)

G2(RB.J)= 0.

DU 100 I= 1.N

1)0 G2(I.J)= G2(I.J)+ V*ZB(I)

G2(RB.J)= -G2(RB.J)

110 CONTINUE

115 V= EB(RB)
o72.
773.
574 .
575.
                         115
577.
                         578 .
579.
:83.
391 .
 782 .
383.
                                  CALL S
394 .
535 .
                                   END
586 .
587.
```

```
SUBROUTINE UPAKC(11)

IMPLICIT REAL*8 (A-H.O-Z)

INTECER*2 JH(350).DIGNA(952).KINBAS(1302).IDHAS(1302)

INTEGER*2 ISTYPE.LA.LE.IA.IF.PUN.LC(20).IC(800)

DOUBLE PRECISION E(8000)

REAL A(4000).C(800).C(800)
588.
 539 .
90.
 591 .
 592.
                                                          A(4000),C(800),CMIN.COND.ERMAX.SUMINE
 593.
594 .
                          C
                                      CCMMCN DSUN.DPROD.DY.DE.DF.B(350).X(350).Y(350).YTEMP(350).

1A.E.CMIN.CCND.ERNAX.SUMINF.[CNAM(13)2.2).NAME(20).

2NTEMP(20).kINP.ITIN.JTIM.ITINV.JTINV.MSTAT.IO3J.IROWP.IVIN.IVOUT.

3ITCNT.INVERQ.ITRLIM.IEFEZ.JCOLP.NROW.NCOL.NELEM.NETA.NLILEM.NLETA.

4NGELEM.NINF.NUELEM.NUETA.NNEGDJ.NLINES.ISTYPE(350).

5LA(1302).LE(2002).PUN(E).

6IPUNC.NDFGI.NDUAL.NIPIW.IFEAS.IFCRSH

COMMON ITCH.ITCHA.IEPIWT.IFNEG.KOUTP

CUMMEN IA(4000).IE(9000)

COMMON/BLCST/BF1(13).BF2(10).E1(10).E2(13).C.IC.LC
 595.
 596 .
 597 .
598.
599.
.000
601 .
603.
604 .
605.
                          C
                             DO 100 I= 1.NROW
Y(1)= 0.
100 CONTINUE
607.
608.
609.
                         C
                                          LL= LC(11)

KK= LC(11+1) -

DO 200 I= LL,KK

IR= IC(1)
610.
611.
613.
014.
                                           Y(18) = C(1)
615.
                               222 CONTINUE
                         C
017.
                                          RETURN
618.
619.
                                 THE FOLLOWING ROUTINE MAKES THE CHANGES IN THE INDEX SETS NECESSARY EVERY TIME A BASIS CHANGE IS MADE.
620.
                         C
621.
                                          SUBROUTINE BOCNC(S.P)
INTEGER*2 JH(350).DICNA(952).KINPAS(1302).IDBAS(1302)
CUMMUNZINDX2Z JH.DIGMA.KINBAS.IDPAS
INTEGER S.SSAV.R.RSAV
RSAV= KINFAS(R)
023.
624 .
025 .
026 .
627.
                                          H(RSAV) = S
KINBAS(P) = O
KINBAS(S) = RSAV
SSAV= IDBAS(S)
628.
629 .
630 .
031.
032.
                                          DIGNA(SSAV)=
                                          IDBAS(S) = 0
IDBAS(R) = SSAV
033.
034 .
635 .
                                          RETURN
636 .
                                 END
THIS SUBROUTINE CAN DO THREE THINGS DETERMINED BY THE
PARAMETER 'KEY'. IT CAN ADD A COLUMN, REMOVE A COLUMN, OR
BOTH AED AND PEMOVE COLUMNS FROM THE PRIMAL OR DUAL SUPER-
PASIC COLUMNS, DEPENDING ON WETHER KEY IS 1, 2, OR 0, RESP-
ECTIVELY. POI IS 1 OR -1 DEPENDING ON WHETHER A PRIMAL OR
DUAL COLUMN IS BEING ADDED. PO2 IS A SIMILAR FLAG FOR THE
CULUMN BEING PEMOVEC. IS AND JS INDICATE THE PARTICULAR
COLUMN TO BE ADDED OR REMOVED FROM THE GROUP OF COLUMNS
637.
639.
040.
041 .
043 .
644 .
045 .
                                  SPECIFIED BY POI AND PD2.
046 .
                                          SUPPOLITINE SUPERE (KEY. PD1. 15. PD2. JS)
```

047 .

```
IMPLICIT PEAL *8 (A-H.O-Z)
INTEGER *D.PDI.PC2.SS.FF.ZFLAG.RS.P.CD.DDI.BLI.BL2
INTEGER*2 JH(350).DIGMA(952).KINPAS(1302).IDEAS(1302)
INTEGER*2 ISTYPE.LA.LE.IA.IE.PUN.LC(20).IC(80))
DOUBLE PRECISION = (8000)
048 .
 049.
050 .
 051 .
 052 .
                                                                                                       A(4000) .C(EUO) .CMIN .CCND . FRMAX . SUMINF
 653.
 054 .
                                            C
                                                                     COMMON DSLN.DPROD.DY.DE.DP.8(350).X(350).Y(350).YTEMP(350).

1A.E.CMIN.CCND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).

2NTEMP(2D).KINP.ITIM.JTIM.ITINV.JTINV.MSTAT.IOBJ.IROWP.IVIN.IVOUT.

3ITCNT.INVFRO.ITRLIM.IFFEZ.JCOLP.NROW.NCOL.NELEM.NETA.NLELEM.NLETA.
 655.
 056.
 057 .
                                                                    3ITCNT.INVERG.ITPLIM.IFFEZ.JCOLP.NROW.NCOL.NELEM.NETA
4NGELEM.NINF.NUELEM.NUEIA.NNEGDJ.NLINES.ISTYPE(350).
5LA(1302).LE(2002).PUN(8).
6TPUNC.NDEGI.NDUAL.NIPIW.IFBAS.IFCPSH
COMMEN ITCH.ITCHA.IFPIWT.IFNEG.KOUTB
COMMEN IA(4000).IF(8000)
CC4MCN/LP1/PI(1302).XX(1302)
CO4MCN/LNCLNE/SI(350.10).G2(400.10).FA(350).88(400)
COMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
ICOMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).NU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MU(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MUH(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MUH(10).MU(10)
COMMCN/INCXI/NUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10).MUH(10)
 058.
 659 .
 000.
 061 .
 663.
 064 .
 065.
006.
 057.
 608.
669 .
677.
                                                                        IF (KEY.EQ.1) GO TO 50
IF (PD2.GT.)) GO TO 20
KM= KMU
KMU= KMU- 1
IF (JS.GT. IH) GO TO 4
MH= MUH(JS)
MUH(JS)= 0
IF (MH.EC.KM) GO TO 6
DO 5 I= MH.KMU
IM= MU(I+1)
MUI(I)= IM
 671 .
 672 .
673.
674 .
675.
 076 .
677.
6/9.
680 .
                                                                          = ( I ) UM
                                                                                                         IM
 081 .
                                                                              MUH([N] = NUH([M)- 1
                                                                         DU 5 J = 1 \cdot N
G2(J.1) = G2(J.1+1)
082 .
683.
                                                                5 CONTINUS
084 .
                                                              6 MU(KM)= 0
4 DU 7 J= 1.N
7 G2(J.KM)= 0.0
IF (KEY.EG.2) RETURN
GO TO 50
 685 .
 086 .
 697 .
098 .
089.
                                                         2) KN= KNU

KNU= KNU- 1

IF (JS .GT. IH) GO TO 28

NH= NUH(JS)
693.
091.
692 .
 694 .
                                                                          NUH(JS) = C
                                                                        IF (NH.:0.KN) GO TC 26
DO 25 I= NH.KNU
JN= NU((+1)
 695 .
096 .
 097.
                                                                         NU (1) = JN
NU = (NL) HUN - 1
 698 .
 699.
                                                           DO 25 J= 1.M
G1(J.I)= C1(J.I+1)
25 CONTINUE
  700.
 701.
                                                           26 NU(KN) = 0
                                                           30 G1(I.KN)= 0.0
  704 .
                                                                                                                          0.0
 705.
                                                                                       (KEY.EG.2) FETURN
 706 .
 707.
                                            C
```

```
NOW WE ACC THE SUPEREASIC VARIABLE SPECIFIED BY POI. IS.
708.
709.
710.
                      50 IF (PDI.LT.0) GD 1C 7C
711.
                            KN= KNU
KNU= KNU+ 1
                            IF (15.51.1H) GO TO SE
NU(KNU) = 15
NUH(15) = KNU
113.
714 .
 115.
                      SS CALL UNPACK(IS)
CALL FIRAN(I)
DO 60 IT 1.M
71 E .
717 .
718.
                      63 GI(I.KNU) = -Y(1)
720 · 721 · 722 · 723 ·
                               NOW WE ARE ERINGING A DUAL SUPERPASIC COLUMN IN.
 724%
                      70 KMU= KMU+ 1
725.
726.
727.
724.
                            IF (IS.61.1H) GO IO 73
MU(KMU)= IS
MUH(IS)= KMU
                      73 IF (IS.LF.M) GO TO 110
724.
                      THE ENTERING VARIABLE IS A ZETA. THE ENTERING COLUMN IS A ROW OF INV(B) AND A FCW OF INV(B)*0.
731 •
731 •
732 •
733.
                            IDD= KINEAS(IS)
                      75 Y(1)= 0.
Y(100)= 1.
734 .
 735 .
730 . 737 .
                           Y(IDD)= 1.

CALL BTHAN

CALL SHIFTR(3.2)

DO 8) I= 1.M

IF (IDEAS(I).EQ.0) GO TO 82

G2(IDBAS(I).KNU)= X(I)
138 .
139.
740 .
742 .
                      80 CONTINUE
                           OS 100 J= NP1.NM

IF (ICEAS(J).EQ.O) GO TO 100

CALL UNPACK(J)

D3T= 3.1
 143.
144 .
145 .
746.
                      DD 90 I= 1.M

IF (ICEAS(I).EQ.O) GO TO 90

DOT= DCT+ Y(I)*X(I)

90 CONTINUE
748.
149.
750.
151 .
                            GZ(IDBAS(J), KNU) = DCT
152 .
                     111 CUNTING
753.
                           RETURN
154 .
                     THE ENTERING VARIABLE IS A PI. THE ENTERING COLUMN IS PARTLY C1*INV(8) AND PARTLY C2+ C1*INV(8)*D.
-C1*INV(8) IS PART OF THE BASIS INVARSE CORRESPONDING TO THE X-BASIC COLUMNS AND THE Y-PASIC ROWS.
755.
756 · 757 · 758 · 759 ·
 160 .
                    110 100= KINHAS(15)
                    00 130 I= 1.M
130 Y(1)= 0.
Y(100)= 1.
 761 .
762.
763.
                           Y(100)= 1.

CALL STPAN

CALL SHIFTR(3.2)

DO 14) I= 1.M

IF (IDEAS(I).EQ.0) GC 10 140
704 .
 165.
157.
```

```
G2(IDEAS(I), KMU) = X(I)

140 CONTINUE
D0 160 J= MP1.NM
IF (ICEAS(J).EQ.0) GC TO 160
CALL UNPACK(J)
D0T= 0.0
D0 150 I= 1.M
IF (IDEAS(I).EQ.C) GO TO 150
D0T= DCT+ Y(I)*X(I)

150 CONTINUS
 760.
769.
 770.
 171 .
 772.
773.
 174 .
 775.
776 · 777 · 778 ·
                                                      150 CONTINUE
                                                                         G2(IDBAS(J).KMU) = Y(IS)+ DOT
                                                      160 CONTINUE
  780.
                                                                          RETURN
781 .
                                                                         END
                                                                          SUBROUTINE RECALC
 782 .
 783.
                                           (
                                                          THIS SUPERUTINE RECALCULATES THE SUPERBASIC COLUMNS USING THE CURRENT BASIS IN OR FORM. IT CAN ALSO ADD A VARIABLE AND COLUMN TO THE SUPERBASICS.
 784 .
 185.
 786 .
 787.
 788.
                                                                           IMPLICIT REAL *8 (A-H.C-Z)
                                                                         INTEGER #0. PD1. PC2. JS. RF. ZFLAG. RS. P. CD. DD1. BL1. BL2
INTEGER #2 JH(350). DIGMA(952). KINBAS(1302). IDBAS(1302)
INTEGER #2 ISTYPE. LA. LE. IA. IE. PUN. LC(20). IC(80))
DOUBLE PRECISION E(8300)
 789.
 790 .
 791 .
 192 .
 793.
                                                                                                    A(4000).C(800).CMIN,COND.ERMAX.SUMINF
                                                       COMMON DSUM.DPROD.DY.DE.DP.B(35)).X(350).Y(350).YTEMP(350).

1A.E.(MIN.CCNE.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).

2NTEMP(20).KINP.ITIM.JITIM.YITINV.MSTAT.IOBJ.IROWP.IVIN.IVDUT.

3ITCNT.INVERG.ITRLIM.IFEEZ.JCOLP.NROW.NCOL.NELEM.NETA.NLELEM.NLETA.

4NGELEM.NINF.NUELEM.NUETA.NNEGDJ.NLINES.ISTYPF(350).

5LA(1302).LE(2002).PUN(8).

6IPUNC.NDECI.NDUAL.NIPIW.IFEAS.IFCRSH

COMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTR

COMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTR

COMMON.LPIPI(1302).XX(1302)

COAMON.LNCCNS.G1(350.10).G2(400.10).EA(350).FB(400)

COMMON.LNDXIZ.NUH(16).MUF(10).NU(10).MU(10)

COMMON.ZCALZ.BT.NR.JJ.MFLAG.DDI.P.PD.MPD.INEG.KEUN.KJAC

COMMON.ZCALZ.BT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.NR.JT.N
 794 .
 195.
 796.
 198.
 199.
 .006
301 .
. 508
833.
804.
a)5.
HC0.
607 .
 109.
 810.
H11.
 H12.
 n13.
 14.
516.
517.
 018.
                                                          28 CONTINUE
                                                          26 CALL SHIFTR(1.3)
CALL FHAN(1)
DO 22 I= 1.M
22 BA(I)= Y(I)
 319.
450 ·
 H21.
                                                          3) IF (KMU.EG.0) GO TO 143
00 130 J= 1.KMU
10D= MU(J)
 374 .
825.
                                                                                     (IDC.LE.M) GO TO 70
 327 .
                                           (
```

```
828.
629.
                               ZETA- VARIABLE IS ENTERING
                   C
                         18= KINHAS(18D)

M,1 = 1 = 1

Y(1)= 1.

Y(1H)= 1.
H31.
H32.
H33.
                               Y(IH)= 1.

CALL BHIFTR(3.2)

DD 40 I= 1.M

IF (IDEAS(I).E0.0) GD TC 40

G2(IDEAS(I),J)= X(I)
n 14 .
H 35 .
836 .
837 .
838.
                         G2(IDBAS(I),J)= X(I)

40 CONTINUE

D0 60 K= MPI.NM

IF (ICBAS(K).EQ.0) GD TO 60

CALL UNPACK(K)

DOT = 0.0

D0 50 I= 1.M

IF (ICBAS(I).EQ.0) GD TC 50

DOT = DCI+ Y(I) * X(I)

50 CONTINUE

G2(IDBAS(K).I)= CDT
H39.
44).
841.
m43 .
844 .
d45 .
846 ·
848 .
                                GZ(IDBAS(K).J)= COT
849.
                         60 CONTINUE
H50 .
                               GO TO 130
851 .
                               PI- VARIABLE IS ENTERING
852.
M53.
                        70 IB= KINHAS(IDD)
DD 90 I= 1.M
90 Y(I)= C.
Y(IB)= 1.
CALL BIMAN
CALL SHIFTE(3.2)
DD 10) I= 1.M
IF (ICPAS(I).FO.C) GD 10 100
G2(IDBAS(I).J)= X(I)
d54 .
855.
456.
457.
M5H.
H59.
860.
                                G2(1)6A5(1).J)= x(1)
n62 .
463.
                       100 CONTINUE
                               CONTINUE
DD 120 K= MP1.NM
IF (IDEAS(K).E0.C) GD TC 120
CALL UNPACK(K)
DDT= 0.0
0.0 110 I= 1.M
IF (IDEAS(I).E0.0) GD TD 110
DDT= DCT+ Y(I) *X(I)
M54 .
865.
866 .
667 .
868.
670.
                      H71 .
M72.
873 ·
H75 .
876.
87H .
879 .
H80.
H81 .
882.
883.
884 .
8d5.
Bist.
n37 .
```

```
888
                                                                       DOT = DCT + X(1)*Y(1)
889.
                                                   165 CONTINUE
88(ICBAS(J)) = DOT
.0es
d91 .
                                                    170 CONTINUS
892 .
893.
                                                                       END
                                                                       SUBROUTINE ENDPNT(JS.PC1.IS.NET)
IMPLICIT REAL*8 (A-H.O-Z)
REAL*8 NIN.MIN2
M94 .
895.
                                                                    REAL*3 NIN.MIN2
INTEGER*2 ISTYPE.LA.LE.IA.IE.PUN.LC(20).IC(80))
INTEGER*2 JH(35).DIGNA(952).KINBAS(1302).IDBAS(1302)
INTEGER PD.PDI.PD2.SS.RF.ZFLAG.P.DD.DD1.BLI.BL2
REAL C(300)
COMMON.NEWIZ H(10,11).X(10).Z(10).ACC(3.10)
COMMON.NEWIZ H(10,11).X(1302)
COMMON.ZHLCSIZBF1(10).EF2(10).E1(10).E2(10).C.TC.LC
CCMMCN.ZBLCSIZBF1(10).EF2(10).E1(10).F2(9.10)
COMMON.ZNCENSZGI(250.10).G2(400.10).EA(350).HB(400)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10)
COMMON.ZNCENSZGI(250.10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH(10).NUH
896 .
 M +7.
899 .
699.
 900.
901 .
902.
 404 ·
 405.
906.
408 .
 . POP
910.
411.
                                                                      DIMENSION U(3).VI(1)), VZ(10).F(10), DOT(4).RHS(10).UL(10.10)
912.
 913.
                                                      THIS PREGRAM WILL FIND THE ENDPOINT DE A CURVE DEFINED BY A SYSTEM OF DD-1 BILINEAR EQUATIONS IN DO UNKNOWNS. CHE ENDPOINT IS KNOWN, AND THE OTHER ENDPOINT IS DETERMINED BY THE FIRST INTERSECTION OF THE CURVE WITH CHE OF NM+1 INEQUALITY CONSTRAINTS.

THE SYSTEM IS
 914 .
415.
916.
 918.
920 .
                                                       BF1+ 02*X(MU)- DIAG(X(MU))*(F1*X(NU)+ E1)= 0
BF2+ D2*X(MU)- DIAG(X(MU))*(F2*X(MU)+ F2)= 0
 421.
 422 .
 423.
                                                        WHERE ONE OF THE ABOVE IS AN INEQUALITY IF ZFLAG.NE.1/
THE INEQUALITY IS THE LAST IN THE GROUP DEFINED BY INEQ.
THE LINEAR INEQUALITIES APE DEFINED BELCW.
 ¥24 ·
 925.
426 .
                                          C
 428 .
                                                                      NET= 0
                                                                       FRAC=1.
429.
                                                                               MFLAG=0 WHEN WE ARE SOLVING FOR THE INITIAL 3 POINTS
MFLAG=1 WHEN CRE OF THE G-CONSTRAINTS IS BINDING.
MFLAG=2 WHEN ONE OF THE VARIABLES GOES TO ZERO.
MFLAG=3 WHEN WE USE THE NORMAL HYPERPLANE TO DEFINE THE
SUBPROBLEM SPORT OF THE HOUNDARY OF THE CELL.
933.
                                          C
931.
432 .
 433.
934 .
                                                                       ITER= 0
936.
936.
937.
                                                                        ISGCT=
                                                                        MFL AG = 0
438 .
                                                                      L=1
439.
                                                                       KK= 0
                                                                       DD1 = DD- 1
440 .
                                                                       ID= ING
941 .
                                                                       KMU1= KMU+ 1
                                                                       KMU1= KMU+ 1
IF (INEG .60. 2) ID= INP+ KMU
IF (F0.EG.C) ID= DD
ICX= ID
IF (KMU.FC.0) GD TD 615
443.
944.
                                                                       DO 610 I= 1.KMU
```

```
610 X(I)= FI(MU(I))
615 IF (KNU.EC.O) GD TO 625
DD 620 I= 1.KNU
KM= KMU+ 1
023 X(KM)= XX(NU(I))
625 ALPHA= -1.
5 ITER= ITER+ 1
IF (DD.6T.1) GO ID 6
KDET= I
GO IC 41
6 IF (KNU.EC.DD .DR. KMU.FC.DD) GO TO 630
CALL DEPIV
7 15GCT= ISGCT+ 1
IF (ISGCT.GT.DD) GC TC 300
DD 10 I= 1.DD1
RHS(I)= -H(I.ICX)
10 CUNIINU-
  943.
   450 .
   951 .
   952.
   954 .
   955.
   956 .
   457.
   459.
   460.
   961.
   963.
   454 .
                                   1) CONTINU-

IF (ICX.EG.DD) GC TC 25

DD 24 J= ICX.DD1

JF1= J+ 1

DD 25 I= 1.DD1

+(I.J)= H(I.JP1)

24 CONTINU

25 IF (DD1.GT.1) GD TD 35

IF (DABS(H(I.I)) .GE. TOLCV) GD TD 26

ISING= 1
   965.
   966.
   968.
   459.
   971.
971.
972.
                                                     DABS(H(1...

ISING= 1

GC TO 37

V2(1)= RHS(1)/H(1.1)

IF (H(1.1).CE.0) CC TC 33

KDET= -1

GO TO 40
   474.
   975.
976.
                                    26
   977.
                                  478.
   479.
   400 .
   481.
   484 .
  495 .
   486.
   987.
   488.
  489.
   990.
  491 .
   493.
                                   39 CONTINUE
                                39 CONTINUE
GU TO 41
630 KDET= 1
DO 634 I= 1.DD
V2(1)= 0.0
634 CUNIINUE
41 V2(ICX)= 1.0
IF (ITER.GT.1) GC TO 45
SUM= 0.0
IF (PD.FG.0) GU TO 120
IF (PD.FG.-1) GO TO 110
JB= KIN3AS(JS)
IF (JB.NF.0) GO TO 104
IF (V2(ID).GI.C) GO TO 155
   994 .
   495.
   996.
   997.
   498 .
1000.
 1001 .
 1002.
1003.
1004.
1005.
1006.
1007.
```

```
GG TU 140

104 DU 105 I= 1.KNU

IK= KMU+ 1

SUM= SUM+ V2(IK)*G1(JP.I)
100H.
1009.
1010.
1011.
                   105 CONTINUE
                  105 CONTINUE

IF (SUM.LT.0.0) GO TO 140

GO TO 135

11) JB= ICEAS(JS)

IF (JB.NE.0) GO TO 112

IF (V2(ID).GT.0) GO TO 135

GC TO 140

112 DO 115 I= 1.KMU

SUM= SUM+ V2(I)*G2(JB.I)

115 CONTINUE
1013.
1014.
1015.
1016.
1018.
1019.
1020.
1021.
                   115 CONTINUS
                         IF (SUM.LT. 0.0) GC TC 140
GO TO 135
1024 .
1025.
                      CALCULATE THE GRADIENT OF THE SURFACE. F(DD) = 0.0.
1026.
             C
                  120 IF (INEO.EO.2) GC TO 126
F(DO) = BF1(ING) + C1(ING..)*PI(MU) - PI(ING)*
1027.
               C
1028.
                                                          (F1(ING..) *XX(NL)+ F1(INQ)).
1029.
                        INO= MUH(DD)
DO 122 I= 1.KMU
V1(I)= D1(ING.I)
1030.
1031 .
1032.
                        DG 123 J= 1.KNU

JK= J+ KMU

SUM= SUM+ F1(ING.J)*X(JK)

V1(ING)= V1(ING)- SUM- E1(ING)

PID= X(ING)

DG 124 J= 1.KNU
1033.
1034 .
1035.
1036.
                        PID= X(INC)
DO 124 J= 1,KNU

    JK= J+ KMU
    V1(JK)= - PID*F1(ING.J)
SUM= 0.0
DO 125 I= 1,DD
    SUM= SUM+ V1(I)*V2(I)
IF (SUM.LI.)) GO TO 140
GO TC 135
1037.
1038 .
1039.
1040.
1042 .
1043.
1044 .
                         GO TC 135
1045.
1046 .
1047.
                     F(DD) = BF2(INQ) + D2(KNU..)*PI(MU) - X(IPQ)*(F2(INP..)*PI(MU)+E2)
                  1048.
1049.
1050.
1051 .
1052.
1054.
                  128 CONTINUE
1055.
1050.
1057.
1058 .
1059.
1050.
1062.
1003 .
1065.
1006.
1067.
```

```
DD 47 1= 1.DD
ACC(2.1) = V2(1)*S1
ACC(3.1) = X(1)
1.08.
1069.
1070.
                         WRITE (6.850)
WRITE (6.855)
1071.
1072.
1073.
                                                              ((ACC(1.J).1=2.3).J=1.DD)
1074 .
                               FIND THE FIRST BOUNDARY THAT THE TANGENT. O(ALPHA) HITS. WE ARE TRYING TO FIND THE SMALLEST ALPHA SUCH THAT G(1,\cdot)*O(ALPHA) + GE(I) = 0
1075.
1076.
                    C
1073.
1079.
1080.
                                IFLAG= 0
                               IF (KNU.EC.^) GO TC 249
DO 250 1= 1,M
IV= JH(I)
1081 .
1042.
1033.
                                IF (IV-FO.M .OR. IV-LE.IH) GO TO 250

DC 240 LL= 2.3

U(LL)= 3.0

EC 230 K= 1.KNU

KJ= K+ KMU
1085.
1086.
1088.
                                                       U(LL)= U(LL)+ G1(I.K)*ACC(LL.KJ)
1089.
1090 .
                                                CENTINUE
                        230
                                       CONTINUE
U(3) = U(3) + EA(1)
IF (DABS(U(2)).LT.TCLFZ) GD TO 250
1091 .
1092.
                        241
1093.
                                       IF (DABS(U(2)).LT.TCLFZ) GO TO 250
ALPHA = -U(3)/U(2)
IF (ALPHA .LT. -TGLFZ) GO TO 250
IF (ALPHA .GT. ICLBD) GC TO 244
TCC= 0.0
DC 242 K= 1.KNU
KN= K+ KNU
TCD= ICD+ G1(I.K)*ACC(2.KM)
IF (TOD .GT. -TOLFZ) GO TO 250
IF (AG = )
 1004.
1095.
10 16.
1697.
1098.
1099.
1100 .
                        242
1101.
1102.
                        244
                                        IFLAG = 1
1103.
                                       IFLAG = 1
JJ= I
MIN= ALPHA
MPD= 1
GC TO 250
IF (ALPHA.GE.MIN) GC TC 250
JJ= I
MIN= ALPHA
MPC= 1
ILLE
1104.
1105.
1106.
1137 .
1108.
                        243
1110.
1111.
                        250 CONTINUE
                        249 IF (KMU.EC.3) GO TO 640
DO 259 I= 1.N
IF (DICMA(I) .EQ. M) GO TO 259
1113.
1115.
                                              U(LL) = 0.0

D0 254 K= 1.KNU

U(LL) = U(LL) + G2(I.K)*ACC(LL.K)
1110.
1117.
1118.
1119.
1120.
                        254
                                                CENTINUE
1121.
                        256
                                        CONTINUS
                                       CONTINUE
U(3)= U(3)+ EP(1)
IF (DABS(U(2)).LT.TOLEZ) GO TO 259
ALPHA= -U(3)/U(2)
IF (ALPHA .LT. -TCLEZ) GO TO 259
IF (ALPHA.GT. TCLED) GC TO 258
1122.
1124 .
1125.
1126 .
1127.
                                               TCC= 0.0
```

```
1128.
                                                                      DC 252 K = 1.KMU
TOD= TOD+ ACC(2.K)*G2(1.K)
1129.
                                                           CENTINUE

IF (TOD .GT. -TCLFZ) GO TO 259

IF (IFLAG.FO.1) GO TO 253
                                   252
1131 .
1132.
                                   259
                                                         IF (IFLAG. FU.T)

IFLAG = 1

JJ = 1

MIN = ALPHA

MPD = -1

GD TD 259

IF (ALPHA.GE.MIN) GC TD 259
1133.
1134 .
1135.
1136.
1137.
1138.
                                   253
                                                           JJ= I
MIN= ALPHA
MPD= -1
1139.
1140.
1141.
                                   259 CUNTINUE
640 MFL AG= 1
1MAG= 0
00 58F I
1142.
1143.
 1144.
                                              IMAG= 0
DD 5H# I= 1.4
    DDT(I)= 0.0
IF (INEQ.EG.2) GC TO 6CO
DD 592 I= 1.DD
    IF (I.GT.KMU) GO TO 591
    DGT(I)= DOT(I)+ C1(ING.I)*ACC(3.I)
    DGT(2)= DOT(2)+ D1(ING.I)*ACC(2.I)
    GC TC 592
    IK= I- KMU
    DGT(2)= DOT(3)+ F1(ING.IK)*ACC(2.I)
    DGT(4)= DOT(4)+ F1(ING.IK)*ACC(2.I)
CONTINUE
1145.
1146.
                                   588
                                   590 IF
1148.
1149.
1150.
1151.
                                   591
1153.
1154.
1155.
1156.
                                   592 CONTINUE
                                              U(1) = -ACC(2, INQ)*DOT(4)

U(2) = -(ACC(2, ING)*E1(ING) + ACC(3, ING)*DOT(4)) + DOT(2)

U(3) = BF1(INQ) + DOT(1) - ACC(3, INQ)*(DOT(3) + F1(INQ))

CALL GUADS(U, IMAG, ALFHA, BF IA)
1157.
1158.
1160.
                                  CALL GUADS (U, IMAG, ALPHA, BF IA)

IF (ALPHA, LT, TOLCV) ALPHA = PETA

GO TO 605

600 IB= INP+ KMU

IF (KMU, EC.2) GO TO 603

DO 602 I = 1, KMU

DOI(1) = DOT(1) + C2(INP, I) *ACC(3, I)

DOI(2) = DOT(2) + D2(INP, I) *ACC(2, I)

DOI(3) = DOT(4) + E2(INP, I) *ACC(3, I)
1161.
1162.
1163.
1164 .
1165.
1166.
1167.
1168.
1169.
                                                             DOT(4) = DOT(4) + F2(INF. 1) *ACC(2.1)
1170.
1171.
1172.
1173.
                                 602 CONTINUE
603 U(1)= -ACC(2.IB) *DOT(4)
    U(2)= -(ACC(2.IB) *E2(INP)+ ACC(3.INP)*DOT(4))+ DOT(2)
    U(3)= BE2(INP)+ COT(1)- ACC(3.INP)*(COT(3)+ F2(INP))
    CALL GLACS(U,IMAC,ALPHA.BETA)
    IF (ALPHA.LT.TOLCV) ALPHA= FETA
605 IF (IMA).EG.1) GC TO 260
    IF (BETA .LI. -TCLCV) GO TG 260
    IF (ALPHA.LT.TOLCV) ALPHA= BETA
    IF (ALPHA .CT. -TOLCV) ALPHA= BETA
    IF (ALPHA .GT. MIN) GC TC 260
    JJ= M+ 1
    MIN= ALPHA
    MPD= 0
                                   602 CONTINUE
1174.
1174.
1175.
1176.
1177.
1178.
1179.
1180.
1181 .
1182 .
                                               MPD= 0
1183.
                                              NOW CHECK TO SEE IF ONE OF THE VARIABLES GUTS TO ZERO BEFORE ANY OF THE CONSTRAINTS BECOME TIGHT.
1134.
1186 .
1187.
                                  260 MFLAG= 1
```

```
DD 280 I=1.DD
DO 265 LL=2.3
U(LL)= ACC(LL.1)
1188.
1193.
                               265 CONTINUE

IF (DABS(U(2)) .LT. TOLFZ) GO TO 280

ALPHA= -U(3)/U(2)

ALPHA= -U(3)/U(2)
1191.
1193.
                                          JJ = 1
MFLAG= 2
1194.
1196 .
                              MFLAG= 2
MIN= ALPHA

280 CGNTINUE
WRITE (6.635) MIN. MPD. JJ
e35 EGRMAT(' STEPLENGTH= '.F15.6.'
IF (MIN.LI. STEPNX) GC TG 255
MIN= MIN* STPRD
MFLAG= 3
WRITE (6.635) MIN. MPD. JJ
255 RI= 0.0
1197.
1198.
1199.
1290.
                                                                                                                             CONSTRAINT TYPE: ( * . 13 . * . * . 13 . * ) * )
1201.
1202.
1233.
1204.
                              255 BT = 0.0

00 290 1 = 1.00

x(1) = MIN * ACC(2.1) + ACC(3.1)

BT = BT + x(1)*ACC(2.1)
1205.
1206.
1207.
1208.
1208.
1209.
1210.
1211.
1212.
1213.
1214.
                        CC
                                    NEWTON'S METHOD FOR THE PROBLEM OF FINDING THE INTERSECTION OF THE CURVE WITH THE CONSTRAINT DEFINED BY MPD AND JJ.
                               295 KK= KK+ 1
                                         IF (KK.GE.5) STOP
DO 100 K= 1.15
KM1= K- 1
1216.
1217.
1218.
                                                       IRWDIM= DD1
                                                   IRWDIM= DD1
IF(MFLAG.GF.1) IRWDIM= DD
CALL FTN(F.X)
CALL NORM(F.S1.IRWDIM)
WPITE (6.54) KM1.(X(I).I=1.DD)
FORMAT (* ITERATION*.13.* X= *.9F10.5)
WRITE (6.936) S1, (F(I).I=1.IRWDIM)
IF (S1.LI.TLLCV) GC TC 800
CALL CERIV
NFI= NET+ 1
IF (DC.GT.1) GO TC 55
IF (DABS(H(I.I)) .LT. TOLCV) GD TO 58
Z(1) = F(1)ZH(1.1)
GC TC 65
CALL DECEMP(IRWDIM.E.UI)
1219.
1219.
1220.
1221.
1222.
1223.
1224.
1225.
1226.
1227.
1228.
1229.
1230.
1231.
1231.
1232.
1233.
                                                                 GC TO 65
DECEMP (TRWDIM . F , UL)
                                                      CALL
                                 55
                                                       IF (1SING.FC.0) GC TO 60
WELLE (6.94C)
1234 · 1235 · 1236 · 1237 ·
                                  58
                                                      STOP
CALL SOLVE( IRWD [N.UL.F.Z)
                                  60
                                                      CALL SGLVE( | RWD | N.UL.F.Z)

ALPH= 1.0

CALL GSENT( ALPH.S1)

IF (ALPH.GT. TOLFZ) SC TO 70

WRITE (6.945)

GC TC 800

CO 95 I= 1.CO

X(I)= X(I)- ALPH * 7(I)
                                  05
1238.
1239.
1240.
1241 .
1242 .
                                 7)
1243.
1244 .
                                95
                               100 CONTINU
1245.
                                         MFLAGE 1
MIN= MIN/2.
1246 .
1247 .
```

```
IF (MIN.GT.IOLBD) GO IC 255
GE TC 300
830 IF (MFLAG.EQ.3) GO TO 5
CALL CONCEK(GMIN.KGMIN.MP2)
IF (KMU.EQ.0) GC TO 146
00 145 I= 1:KMU
KM= NU(I)
1248.
1249.
1250.
1251 .
1252.
1254 .
                                          PI(KM)= X(I)
IF (X(I).GE.CMIN) GC TO 145
GMIN= X(I)
KGMIN= KM
1255.
1256.
1259.
                                                     MP2= -1
                       145 CONTINUE
146 IF (KNU.EC.O) GO 1C 149
DO 147 I= KMUI.DE
IK= I- KMU
1261.
1202.
1263.
1264 .
                                           KM= NU(IK)
                                          KM= NGTN,

XX(KM)= X(I)

IF (X(I).GE.GMIN) GC TO 147

GMIN= X(I)

KGMIN= KM
1265.
1267 .
1268.
1269 •
1270 •
1271 •
                                                   MF2= 1
                        147 CUNTINUE
                        148 IF (GMIN .LT. -TCLBD) GG fC 170
IF (MPD .NE. 0) GG TO 153
1272.
1273.
1274.
                                 PD1= 0
                                RETURN
                        153 IF (MFLAG.EQ.2) 60 TC 150
IF (MPD.EG. -1) 60 TO 151
IS= JH(JJ)
POI= 1
1275 .
1276.
1277.
1279.
1279.
                                RETURN
                        151 IS= DICMA(JJ)
1281 .
1282.
                                 RETURN
                        150 IF (JJ.61.KMU) GC TO 152
15= MU(JJ)
1283.
1284 .
1285.
                                PD 1 = -1
                        152 JJ= JJ- KMU
152 JJ= JJ- KMU
15= NU(JJ)
P01= 1
1286.
1287.
1288 .
1289.
1290 .
                                RETURN
1291 .
                           IF THE CURRENT VALUE OF X VIOLATES THE CONSTRAINT DEFINED BY KGMIN AND MP2. FIND A GOOD STARTING POINT FOR NEWTON'S METHOD BY FINDING THE POINT ON THE LINE SEGMENT ACC(1..) + ALPHA*(X - ACC(1..)). O<= ALPHA<= 1 THAT SATISFIES THE VIOLATED CONSTRAINT EXACTLY.
1292 .
1293.
1294 .
1275.
1296.
1298 .
                        170 JJ= KGNIN
1299.
                                MPD = ME2
1300.
                                MFL AG= 1
                       WPITE (6.174) MPC.JJ.GNIN

174 FURMAT(/, THE QUADRATIC PICKED THE WRONG CONSTRAINT.'./

1. THE NCST INFEASIBLE CONSTRAINT IS TYPE '.I3.

2. NUMBER '.I4./, WITH A VALUE OF '.FI4.6)

DO 171 I= 1.DC
1.501 .
1.612.
1 504 .
                        171 DO 171
1305.
                                V1(1) = X(1) - ACC(3.1)
DO 175 1= 1.4
1306 .
1.07.
```

```
DC1(I)= 0.0

IF (MPD.EC.0) GD TC 190

IF (MPD.EC.-1) GC TU IFO

DD 172 I= 1.KNU

IK= I+ KMU

DCT(I)= DOT(I)+ CI(JJ.I)*ACC(3.IK)

DCT(2)= DOT(2)+ GI(JJ.I)*VI(IK)
1308.
                           175
1.309.
1.110 .
1.311.
1.312.
1313.
1.314 .
                           172 CONTINUE
1315.
                           1.16.
1.317.
1318.
1319.
1320.
                                              DGT(1)= DGT(1)+ G2(JJ.I)*ACC(3.I)
DCT(2)= DGT(2)+ G2(JJ.I)*V1(I)
1322.
                          183 CONTINUE

STR= (-EE(JJ)-DOT(1))/DOT(2)

DO 196 T= 1.DD

X(1)= ACC(3.1)+ STR*V1(1)
1324 .
1325.
1326.
1.,28.
                               THE BILINEAR CONSTRAINT IS NOT SATISFIED AT X. FIND A GOOD STARTING POINT BY SOLVING THE QUADPATIC DEFINED BY THE INTERSECTION OF THE SEGMENT DEFINED ABOVE AND THE CONSTRAINT.
1329.
1000.
                      C
1331.
1333.
                           19) IF (INEO.EG.2) GC TO 200
1334 .
                                   DO 192 T= 1.00
IF (1.GT.
                                               TF (I.GT.KML) GO TO 191
DGT(1)= DGT(1)+ C1(ING.I)*ACC(3.I)
DGT(2)= DGT(2)+ D1(ING.I)*V1(I)
1335 .
1336.
                                              DCT(4)= DOT(4)+ F1(ING.IK)*ACC(3.I)
DCT(4)= DOT(4)+ F1(ING.IK)*V1(I)
1338 .
1339.
                           191
1.540 .
1341 .
                           192 CONTINU
1.542 .
                                   U(1) = -V1(INO)*DCT(4)

U(2) = -(V1(ING)*E1(ING)+ ACC(3.ING)*DOT(4))+ DOT(2)

U(3) = BF1(ING)+ DOT(1)- ACC(3.ING)*(DOT(3)+ E1(ING))

CALL CU4DS(U.IMAG.STH.EETA)
1343 .
1.344 .
1 145 .
1.46 .
                                  IF (STR.LT.TOLCV) STR= PETA
DO 193 [= 1.00
X(1)= ACC(3.1)+ STR*V1(1)
GO TO 295
1347 .
1.348 .
                           193
1349 .
1350 .
                          GO TO 295

200 IB= INP+ KMU

IF ( KMU.EQ.O) GC TO 204

DU 202 I= 1.KMU

DOT(1)= DOT(1)+ D2(INP.I)*ACC(3.I)

DCT(2)= DOT(2)+ D2(INP.I)*V1(I)

DCT(3)= DOT(3)+ F2(INP.I)*ACC(3.I)

DCT(4)= DOT(4)+ F2(INP.I)*V1(I)
1351 .
1352 .
1.153.
1354 .
1355 .
1.156.
1.157.
                          202 CUNTINUE
1358 .
                          202 CONTINUE

204 U(1) = -V1(IE)*DD1(4)

U(2) = -(V1(IH)*E2(INF)+ ACC(3.INF)*OCT(4))+ DD1(2)

U(3) = BF2(INF)+ DCT(1)- ACC(3.INF)*(DDT(3)+ E2(INF))

CALL QUADS(U.IMAG.STR.BETA)

IF (STF.LT.TGLCV) STR= HETA

DD 203 I= 1.DD

203 X(1) = ACC(3.I)+ STR*V1(I)

GO TC 235

300 WRITE (5.580)
1359 .
1.160 .
1362 .
1363.
1364 .
1365.
1.567 .
```

```
850 FORMAT(1X. THE CURRENT LINEAR APPROXIMATION TO THE CURVE IS:
1.568 .
                       1./.1X.' A2(*U)+ A3')

855 FORMAT (1X,2F17.7)

936 FORMAT (1X,*NORM(F(X))= '.F10.6.' F= '.9F10.6)

940 FORMAT (1X.//*****ALGCRITHM BONNED WITH SINGULAP JACOBIAN*****)

945 FORMAT (1X.//*****ALGCRITHM BONNED WITH SINGULAP JACOBIAN*****)
1.569.
1373.
1371 .
1572 .
1.5/4.
                        980 FORMAT(IX. "NEWTON"S WETHED FAILED TO CONVERSE.")
1375.
                                STOR
                                IND
                                QUALS FINDS THE SMALLEST NONNEGATIVE POUT OF U(1)*ALPHA**2 + U(2)*ALPHA + U(3) = 0. IF THERE IS ONE. IF NOT. IMAG IS SET = 1.
1.377 .
1378 .
1379.
1380 .
                               SUBROUTINE QUADS (U.IMAG.ALFHA.BETA)
IMPLICIT BEAL *8 (A-H.Q-Z)
INTEGE PD.PD1.PD2.SS.ER.ZELAG.P.DD.CD1.BL1.EL2
INTEGE P2. J4(350).DIGMA(952).KINBAS(1302).IDBAS(1302)
CUMMCNZTULERZ TOLEZ.TOLED.TCLCV.STPNX.STPRD
CCMMCNZDINZ IH.N.M.GD.KMU.KNU.EL1.EL2.MP1.NM.INQ.IND
1.191 .
1332 .
1383.
1.544 .
1345 .
1.386 .
                                DIMENSION U(3)
                                IMAGE C

IF (DABS(U(1)).GI. TOLFZ) GC TO 1)

IF (DABS(U(2)).LI.TGLEZ) GC TO 8
1.188.
1389.
1390 .
                                ALPHA = -U(3)/U(2)
BETA= ALPHA
RETURN
1391 .
1392 .
1.03.
1.594 .
                            8 IMAG= 1
                               RETURN
1375.
                          1) RT= U(2)**2 - 4*L(1)*U(3)
IF (RT) 20, 30, 40
1396 .
1.197 .
1.598 .
                                  THE GUADRATIC DEES NOT INTERSECT THIS CONSTRAINT.
1399.
                   C
1430 .
1401.
                          20 IMAG= 1
1402.
                               RETURN
                                THERE IS A DOUBLE FUCT.
1404 .
1405.
1400 .
                          30 ALPHA = U(2)/(U(1)*2.)
BETA= ALPHA
14)7.
1408.
                                PETUPN
1419.
1410.
                                  TWC REAL ROOTS EXIST.
1411.
                         40 DIS= DSQFI(PT)
ALPHA= (-U(2)-DIS)/(2.*U(1))
BETA= (-U(2)+DIS)/(2.*U(1))
IF (ALFHA - LE. BETA) RETURN
SAVE= ALPHA
ALPHA = BETA
1412.
1414.
1416.
                                BETA= SAVE
1418.
1419.
                                RETURN
1420 .
1421 .
                                SUBROUTINE CONCER(GMIN.KGMIN.MP2)
1422.
                         THIS SUPROUTINE EVALUATES THE CONSTRAINT FUNCTIONS AND FINDS THE SMALLEST VALUE IN GMIN. ZFLAG IS BOTH AN INPUT AND OUTPUT PARAMATER. IF ZFLAG.EG.1 INITIALLY. THEN ONLY THE NONLINEAR CONSTRAINT IS EVALUATED. ZFLAG IS SET EQUAL TO 2 IF ANY CONSTRAINT IS NONPOSITIVE. OTHERWISE IT
1423.
1424.
                    C
```

```
REMAINS ZEGO.
1428.
1424.
                                      IMPLICIT REAL*8 (A-H.O-Z)
INTEGER PD.PD1.PE2.SS.RF.ZFLAG.P.DD.DD1.HL1.RL2
INTEGER*2 ISTYPF.LA.LE.IA.IE.PUN,LC(2)).IC(8))
REAL A(4000).C(800).CMIN.CGND.EPMAX.SUMINF
INTEGER*2 JH(350).DIGNA(552).KINEAS(13)2).IDBAS(13)2)
CGMMCN.NR.MZ.H(10.II).X(10).X(10).AC(3.I0)
CUMMON.HLCST/BF1(10).BF2(10).E1(10).E2(10).C.IC.LC
CDMMCN.HLCST/BF1(10).BF2(10).F1(9.10).F2(9.10)
CGMMCN/LRCST/BF1(10).B2(5.10).F1(5.10).F2(9.10)
CGMMCN/LRCST/BF1(10,10).B2(400.10).FA(350).B3(400)
CGMMCN/LNCENS/G1(350.10).G2(400.10).FA(350).B3(400)
CGMMCN/INDX2/JH.DIGMA.KINFAS.IDPAS
CUMMUN/SCAL/ HT.NR.JJ.MELAG.DD1.P.PD.MPD.INEO.KFUN.KJAC
CCMMCN/DIM/ IH.N.M.DD.KMU.KNU.BL1.BL2.MP1.NM.INQ.INP
GMIN= 1).
1430 • 1431 • 1432 •
1433.
1434 .
1435.
1436 .
1437 .
1438 .
1419.
1440 .
1441 .
1443.
                                       GM IN= 10.
1444.
                                       ZFLAG=
                                      DO 20 1: 1.M

1K= JH(I)

CG= 0.0
1445 .
1445 .
1447 .
1448.
                                                 IF (KNU.EQ.C) GC TC 15
                                                DD 10 J= 1.KNU
JK= J+ KNU
GG= GG+ G1(1.J)*X(JK)
1449 .
1450 .
1451 .
1452 .
                                                CONTINUE
                               10
                                                GG= GG+ PA(I)

XX(IK)= GG

IF (IK .EC. N) GC IC 20

IF (GG.GE.GMIN) GC TC 20
1453.
1454 .
1455 .
1456 .
1457 .
                                                GMIN= GG
KGMIN= I
1458.
                                                MP2= 1
1460 .
                               20 CONTINUE
1461.
                                      00 40 1-
1462 .
                                                IK = CICMA(I)
1463.
                                                GG= 0.0
IF (KMU.FG.0) 60 TC 35
1404 .
                                                00 (C J= 1.KNU
GE= GG+ G2(1.J)*X(J)
1465.
1+66 .
                                              CENTINUE
1467.
                               30
                                                GG= GC+ FB(I)
1468.
1470 .
                                                      (GG.GE.GMIN) GO TC 40
                                                GMIN= CG
KGMIN= I
1471 .
1472 .
                                                MP2= -1
1474 .
                               46 CONTINUE
                               4C CONTINCT
IF (FD.EG.O) RETURN
45 DOT= C.
DOT1= C.
GG= 0.0
IF (INEO - 2) 50.60.60
50 IF (KNU.EQ.O) GD 16 55
DO 54 J= 1.KNU
JK= J+ KMU
54 DDT= DETA FICTION - 1) *X(IK)
1475 .
1476 •
1477 •
1478 •
1479 .
1440.
1481 .
1482.
                               54 DUT = DUT+ FI(INO.J) + X(JK)
1453.
                               55 GG= BEI(INC) - X(INC)*(DCT+ EI(INO))
IF (KMU.(C.O) GO TO BO
DO 57 J= 1 KMU
 1484 .
1485.
148t.
1497 .
                               57 GG= (G+ C1(INQ.J)*X(J)
```

```
GO TC 83

60 KB2= KMU+ INP
    IF (KMU.EQ.O) GO TO 65
    DO 63 J= 1.KMU
    DOT= DOT+ F2(INP.J)*X(J)

63 OCTI= DOT1+ D2(INP.J)*X(J)

65 GG= BF2(INP.)+ DOT1- X(KP2)*(DOT+ E2(INP))
    GU TO 80

80 IF (GG.GE.GMIN) FETURN
    GMIN= CO
1488 .
1489.
 1490 .
1491 .
 1492 .
 1493 .
  1494 .
 1495 .
 1496.
                                                                                               MHS = C
 1477 .
 1498.
  1499.
                                                                                                RETURN
1500.
1501.
                                                                                       SUBROUTINE FIN EVALUATES THE FUNCTION F(x)=
   (3(MU)+ D1*X(MU) + D1AG(X(ML))*(F1*X(NU))
   (H(NU)+ D2*X(MU) + D1AG(X(NU))*(F2*X(MU)+ F3)
 1503.
1505.
                                                           C
                                                                                              SUBROUTINE FIN(F.Y)
1507.
                                                           C
                                                                         THIS ROUTINE EVALUATES THE FUNCTION WHICH THE ENDPOINT SUBROUTINE IS CURRENTLY TRYING TO FIND A ROOT OF. IF MELAGEOUALS I THEN THE LAST FUNCTIONAL COMPRISING F IS ONE OF THE INEQUALITY CONSTRAINTS-- THE INEQUALITY DETERMINED BY THE PARAMETERS MPC AND JJ.
 1:008.
 1509.
1510.
1011.
1512.
                                                                                            IMPLICIT REAL #8 (A-H.C-Z)
INTEGER PD.PD1.PD2.SS.FR.ZFLAG.P.DD.CD1.BL1.RL2
INTEGEP #2 ISTYPE.LA.LE.IA.IE.PUN.LC(20).IC(800)
REAL A(4000).C(800).CMIN.CCND.ERMAX.SUMINF
INTEGER*2 JH(35)).DIGMA(952).KINEAS(1302).IDBAS(1302)
CGMMCN.MEWIZ H(1C.11).X(10).Z(11).ACC(3.11)
CGMMCN.MEWIZ H(1C.11).X(10).E2(10).E2(10).C.IC.LC
CUMMCN.MECSTZBI (10).EF2(10).E1(10).E2(10).F2(9.10)
CGMMCN.MCCNS.GI (350.10).D2(9.10).F1(9.10).F2(9.10)
CGMMCN.MCNS.GI (350.10).G2(400.10).FA(350).B3(400)
CGMMCN.MCNS.GI (350.10).KG(400.10).FA(350).B3(400)
CGMMCN.MCNS.GI (350.10).C.GI (400.10).FA(350).B3(400)
CGMMCN.MCNS.GI (350.10).FI (400.10).FA(350).B3(400)
CGMMCN.MCNS.GI (350.10).FI (400.10).FA(350).B3(400)
CGMMCN.MCNS.GI (350.10).FI (400.10).FI (400.10).FI (400.10).FI (400.10)
CGMMCN.MCNS.GI (350.10).FI (400.10).FI (400.10).FI (400.10).FI (400.10).FI (400.10).FI (400.10)
CGMMCN.MCNS.GI (350.10).FI (400.10).FI (400.10).
 1514.
1515.
1516.
 1518.
 1519.
 1520 .
 1521 .
 1523 .
 1524 .
 1525 .
1526 .
 1527.
1529.
1529 .
1530 .
                                                                                KMU1= KMU+ 1

190= 0

1F (KMU.FC.0) GO TO 23

DO 20 1= 1.KMU

IF (INEO.EG.2) GC TO 5

IF (I.EG.ING) GO TO 15

5 IR= I- IFO

DOT= 0.)
 1531 .
1532.
1534 .
 1535 .
 1536 .
1537 .
                                                                           DOT= ).)

IF (KNU.EQ.O) GO TO 15

DO 10 J= 1.KNU

JK= J+ KMU

10 DOT= DOT+ F1(I.J)*Y(JK)

15 F(IR)= -Y(I)*(DOT+ E1(I))+ EF1(I)

IF(KNU.EQ.O) GO TO 20

DO 18 J= 1.KMU

18 F(IR)= F(IR)+ D1(I.J)*Y(J)

GO TO 20

19 IRO= 1
1538 .
1539.
1541 .
1542 .
  1543.
 1545 .
1546 .
                                                                             19 180= 1
1547 .
```

```
20 CONTINUE

23 IF (BL2.FC.6) GO TO 70

18= INF+ KMU

OI 40 I= KMU1.00

1MH= I- KMU

1F (INFO.EG.1) GC TC 24

1F (I.EJ.18) GO TO 35

24 IRS I- IFU

OUT= 0.0

1F (KMU. G.0) GO TO 30
1548 .
1549.
1550.
1551 .
1552 .
 1554 .
1555 .
1556 .
                                             101= 0.0

100 25 J= 1.KMU

25 201= DCI+ F2(1MB.J)*Y(J)

30 F(I=)= -Y(I)*(COT+E2(IMB))+ BF2(IMB)

1F (KMU.FC.O) GO TC 42

DU 32 J= 1.KMU

32 F(IR)= F(IR)+ D2(IMB.J)*Y(J)

GO TO 42

35 180= 1
 1557 .
 1958.
1559.
1560 .
 1561 -
1562 .
 1565.
                                             35 INO=
 1564 .
 1365 .
                                              40 CONTINUE
 1566 .
                                             70 1F (MFL A5.LI.1) FFTURN
1F (NPD.AF.0) GC IC 120
                                            1F (NPD.NF.0) GC TC 120

DOT1= 0.

DOT1= 0.

DOT2= 0.

IF (INFO - 2) 80. 90. 90

HO IF (KNU.EQ.0) GO TO 85

DO 34 J= 1.KNU

JK= J+ KML

84 DOT= DCT+ F1(INO.J)*Y(JK)

85 F(DD)= BF1(INQ)- Y(INC)*(DOT+ E1(INQ))

1F (KMU.EQ.0) GO TO 120

DO 87 J= 1.KMU

87 F(DD)= F(CD)+ C1(INQ.J)*Y(J)

GO TO 12)
1500.
1569.
1570.
1571.
1572.
1573.
1574.
 1575.
1575.
1577.
1577.
1579.
                                         87 f(DC)= f(CE) + CI(INO,J)*Y(J)
GO TO 12)
90 TF (KMU.EG.O) GO TC 95
DO 93 J= 1+KMU
DOT= C(T+ F2(INP,J)*Y(J)
93 DOT1= DOT1+ D2(INP,J)*Y(J)
95 IKN= INP+ KMU
f(DO)= REZ(INP)+ DCTI- Y(IKN)*(DCT+ E2(INP))
120 IF (MFLAG.LT.1) RETURN
IF (MFLAG.C.T.1) RETURN
IF (MFD.EG.O) RETURN
1F (MPD.EG.-1) (C TC 14C
IF (KNU.EC.O) GO TO 135
DO 130 J= 1-KNU
JK= J+ KMU
130 F(DC)= F(DC)+ GI(JJ.J)*Y(JK)
135 F(DC)= F(CC)+ BA(JJ)
RETURN
140 IF (KMU.EG.O) GO TC 150
  530.
1581 .
 1582.
1583.
 1534 .
1586.
1587.
1588.
1589.
 1590.
1591.
 1593.
 1594 .
1595.
1596.
1597.
                                         140 LF (KMU.FG.0) GO TC 150

DO 145 J= 1.KMU

145 F(DC)= F(CD)+ G2(JJ.J)*Y(J)

150 F(DD)= F(DD)+ EB(JJ)
1598 .
1599.
 1001 .
 1632 .
                                                       RETURN
                                         160 F(DD) = Y(JJ)
RETURN

170 DOT = 0.0
DO 180 J = 1.DD

180 DCT = DCT + ACC(2.J)*Y(J)
 1003.
111)4 .
1605.
1000 .
1607.
```

```
F(DC)= DGT- BT
1608.
1009.
                                                                   RETURN
1610.
                                                                   END
                                                                    SUMBDUTINE DERIV
1011.
                                                              THIS SUPERCUTINE CALCULATES THE EXACT JACOBIAN OF THE BILINEAR FUNCTION DEFINED BY BLOOMS. THERE MAY OR MAY NOT BE ANOTHER FUNCTIONAL APPENDED WHICH WE ARE ATTEMPTING TO MAKE BINDING WITH NEWTON'S METHOD. ALSO, A COLUMN MAY BE GMITTED IF THE ENDPOINT ALGORITHM FEQUIRES
1013.
 1014.
1015.
                                          C
 1017.
                                                                THE CERESPENDING VARIABLE TO BE FIXED.
1018.
 1019.
                                                                   1620.
1021.
1622.
1023.
 1025.
 1026 .
 1027 .
1628.
 1010.
 1031 .
 1632 .
                                                                     INTECER CELJJ
                                                                   INTECER CCLJJ

KJAC= KJAC+ DD * DD

IF (MFLAG.EG.0) KJAC= KJAC- DD

DO 10 I= 1.DD

H(I,J)= C.
 1033.
1634 .
1035.
 1036 .
 1037 .
                                                                   KMU1= KMU+1
1039 .
                                                                    KNU1= KNU+ 1
1034.
                                                                    DD1 = DD-1
1640 .
                                                                    COLJJ= )
                                                                   COLJJ= J

IF (KMU.EG.O) GO TC 55

DO 5) J=1,KMU

IF (J.EO.JJ .AND. MFLAG.GT.1) GO TO 45

JM= J-CCLJJ
1042.
1043.
1044.
1645.
                                                                    IF (KMU.EC.O) GC TC 35
IRC= C
DO 30 I= 1.KMU
1046.
1647 .
1048 .
                                                                                                      IF (INEQ.EC.2) GO TO 15
IF (I.EQ.INQ) GU TO 28
1649.
1050 .
                                                                                                     19= 1- 180
H(IR.JM) = D1(1.J)
IF (1.NF.J) 60 TC 30
COT= 0.0
                                                                                                       19=
 1651 .
                                                       15
1052 .
1653.
1654 .
                                                                                                      IF (KNU.EQ.0) GC TO 25
DO 20 K= 1.KNU
KH= KMU+K
 1655 .
1050.
1057 .
                                                                                                     DOT= DCT+ F1(I,K)*X(K6)
H(IR,JM)= H(IR,JM)- COT- E1(I)
GC 10 30
IF(= 1
1658 .
                                                       20
 1059.
                                                        25
1060 .
 1661 .
                                                       28
                                                                                     CONTINUE
                                                        30
 1002.
                                                                                    THE TERMULATE TO SC TO S
 1053.
 1054 .
1005.
1667.
                                                                                                       IF (IMB.EG. INP) GC TO 38
```

```
IF= I- IFC
F(IR,JM)= D2(IMB,J)- F2(IMB,J)*X(I)
GE 10 40
IFO= 1
1668.
                                37
1669.
1670.
1671.
                                38
1072.
                                                 CONTINUE.
                                4)
1674.
                                                 GO TO 50
COLJJ= 1
                                45
1075.
                                50 CONTINUE
                               50 CONTINUE

55 IF (KNU.EC.O) GO TO 141

DO 70 J= KNU1.DO

IF (J.EO.JJ..AND. MFLAG.CT.1) GC TO 65

JM= J-COLJJ

JMU= J-KMU

IF (KMU.EC.O) GO TO 72
1677 .
 1078.
1680.
1651 .
1082 .
                                                 IRC =
                                                16 1= 1,KNU

16 (1,E0.EC.2) GC TC 57

16 (1,E0.1NG) GC TC 58

16=1-16C

H(18,JM) = - X(1)*F1(1,JMU)
1683.
1684 .
1686 .
1637.
                                                          GG TO 60
IRC= 1
1688.
1089.
                                                CONTINUE
GO 10 7)
1690 .
                                60
1691 .
                                65
                                                 COLJJ= 1
1692.
                                73 CONTINUE
COLJJ= 0
72 IRG= 0
10+3.
1094 .
1695.
                                       IF (INEG .EQ. 1) IRC= 1
IF (6L2.EC.0) GO TO 141
DO 80 1= 1.KNU
IKM= I+KMU
1096 .
1097 .
1698 .
1099.
                                       IF (IKM.EC.JJ.AND.MFLAG.CT.1) COLJJ= 1
IF (IKM.EC.JJ.AND. MFLAG.CT.1) GO TO 8)
IF (INEQ.EC.1) GC TO 73
IF (I.EC.INP) GO TO 78
IR= I- IEC
1701.
1702.
1703.
1704.
                               73 IR= I- IRC

IK= KMU+IK

KNC= IKM - CCLJJ

IF (KMU.EC.J) GO TC 77

DO 75 K= 1.KMU

75 H(IK.KNC)= H(IK.KNC)- F2(I.K)*X(K)

77 H(IK.KNC)= H(IK.KNC)- F2(I)

GO TO 80

78 IRC= I

20 CONTINUE
1705.
1706.
1707.
1708.
1709.
1710 ·
1711 ·
1712 ·
1713 ·
                                80 CONTINUE
1714.
1715.
1716.
1717.
1718.
                        000
                                  IF MELAGE1 THE INEQUALITY SPECIFIED BY MPD AND JJ
DETERMINES THE DDTH ROW OF THE JACOBIAN MATRIX H.
                                       IF (MFLAG.LT.1) FETURN
1719.
1720.
1721.
1722.
                                       IF
                                               (MPD.FG.0) GO TO 145
                             IF (MPD.FG.0) GO TO 145
GO TO 120

145 IF (INFO-2) 150,160,160
150 DO 155 J= 1.DC
IF (J.GT.KMU) GO TC 157
H(DD.J)= CI(INQ.J)
IF (J.NF. INO) GC TO 155
DOT = 0.0
1/23.
1724 •
1725 •
1726 •
                                             (KNU.FG.0) GO TO 154
1727.
```

```
DO 152 K= 1.KNU

KB= KMU+ K

152 DOT= DOT+ F1(INO.K)*X(KB)

154 H(DD.J)= H(DD.J)- DGT- E1(KMU)

GO TC 155

157 JK= J-KMU

H(DC.J)= -F1(INO.JK)*X(INQ)
172H.
1729.
 1730 .
 1731 .
1732.
1734 .
 1735.
                            155 CONTINUE
                           165 CONTINUE

GO TO 180

160 KB2=1NP+ KMU

DO 164 J= 1.0D

IF (J.GT. KMU) GO TO 165

H(DD.J)= D2(INP,J)- F2(INP,J)*X(KB2)

GO TO 164

165 IF (J.NF.KB2) GO TO 164

DOT= 0.0

IF (KMU.50.2) GO TO 168
1730 .
1737 .
1738 .
 1740 .
1741 .
1742.
1743 .
                            167 DOT = DUT+ F2(1NF,K)*X(K)
168 H(DC,J) = -COT-E2(1NP)
1744 .
 1745.
1746.
 1748 .
                            164 CONTINUE
                            G0 TG 1°2

180 IF (MELAG.LT.1) RETURN

IF (MPC.EG.0) RETURN

IF (MELAG - 2) 190.23C.240

19) IF (MEC.EG. -1) CO TG 210
1749.
1750 .
1751 .
1752.
1/53.
                           DO 200 J= KMU1.88

JM= J- KMU

200 H(DB.J)= G1(JJ.JN)

RETURN
1754 .
1735.
1756.
                           550 H(DE.J)= (5(JJ.J)
1758.
1759 .
                           220 H(DE-J) - (1.2)
RETURN
230 H(DD-JJ)= 1.0
RETURN
240 DD 250 I= 1.DD
250 H(DD-I)= ACC(2.1)
1760 .
1761 .
1/52.
1764.
                                    RETURN
1765.
1756.
                                     END
                                   END

SUBPOUTINE NORM(Y,SI.N)

IMPLICIT REAL *8(A-H.C-Z)

DIMENSION Y(10)

SI = 0.0

DO 10 I= 1.N

SI = SI+ Y(I)*Y(I)

SI = DSCPI(SI)
1707 .
1768.
1769.
1771.
1/72.
1774 .
                                     RETURN
1775.
1777.
                                       DSENT IS A SUBREUTINE WHICH IMPLEMENTS DESCENT ON THE NERM OF THE FUNCTION WHOSE ROOT IS BEING DETERMINED BY NEWTON'S METHOD.
                       C
1779 .
                       C
1780.
1781.
                                     SUBPOUTINE DSENT (ALPHA . FNCRM)
1782.
                                    IMPLICIT REAL*8 (A-H.C-Z)
INTEGER PD.PD1.PD2.SS.RE.ZFLAG.P.DD.CD1.BL1.BL2
CD4MCNZDIMZ IH.N.M.DD.KMU.KNU.BL1.BL2.MP1.NM.INQ.INP
CDMMCNZNEWIZ H(1C.11).X(10).Z(10).ACC(3.10)
DIMENSION Y(10).F(10)
1784 .
1/85.
1786.
1787.
                                1 00 5 1=1.00
```

```
1788.
                        5 Y(I)= X(I)- ALPHA#Z(I)
CALL FIN(F,Y)
CALL NCHM(F,S2.DE)
IF (S2.LT.PNDRM) RETURN
ALPHA= ALPHAZ2.
1790.
1791 .
1792 .
1/93.
                                 (ALPHA.CT. .00001) GO TO 1
                            RETURN
1794 .
1795 .
                            END
1796.
                        DECOME AND SOLVE ARE BORROWED FROM FORSYTHE'S THE SOLUTION OF LINAER SYSTEMS OF EQUATIONS. THEY IMPLEMENT A GAUSSIAN ELIMINATION SCHEME WITH PARTIAL PIVOTING TO PRODUCE A LU DECOMPOSTION OF THE MATRIX A. 'SCLVE' PERFORMS THE BACK SUBSTITIONS NECESSARY TO SOLVE UL * X = B . KDET FINDS SIGN( DET A ).
1797.
1798 .
1799.
1800.
1801 .
1002.
                            SUBROUTINE DECOME(NN.A.UL)
IMPLICIT REAL *8(A-H.C-Z)
COMMONZINIZ IPS(30), KDET, KOUNT. ISING
1803.
1804 .
1835.
1806.
                            DIMENSION A110.101.UL(10.10). SCALES(10)
1837 .
                            N= NN
1808.
                            KDET= 1
1839.
1810.
                         INITIALIZE IPS.UL. AND SCALES.
                            ISING= 0
00 5 [= 1.N
IPS(1)= 1
1811.
1813.
1014 .
                                   RCWNEN = 0.0
1815.
                                   00 2 J= 1.N
                                         UL(I,J)= A(I,J)
IF (RCWNRM-DAES(UL(I,J))) 1.2.2
EGWNRN= CABS(UL(I,J))
1816.
1818.
1819 .
                                   CONTINUE
                                   IF (RCWNRM) 3,4,3
SCALFS(I) = 1./RCWNRM
1820.
1421.
                        3
                                   GO TO
1822 .
                                   CALL SING(1)
1823.
1624.
                                   ISING=
1025.
                                   SCALES(I) = J.
                        5 CONTINUE
1826.
                 C
1828 .
                        GAUSSIAN FLIMINATION WITH PARTIAL PIVOTING
                           NM1= N- I
DO 17 K= 1.NM1
B1G= 0.0
DO 11 I= K.N
1829.
1830 .
1831 .
1832 .
                                         IT = K,N

IF = IPS(I)

SIZF = DAES(UL(IP•K))*SCALES(IP)

IF (SIZE-BIG) 11.11.10

BIG= SIZE
1833.
1834 .
1835.
1836 .
                      10
1837.
                                                      ICXPIV=
1838 .
                      11
                                   CONTINUE
                                         HIG) 13.12.13
CALL SINC(2)
                                   IF (HIG)
1839.
1040 .
                      12
                                          ISING=
1841 .
                                              TO 17
1843.
                       13
                                   IF (IDXPIV-K) 14.15.14
                                          J= IPS(K)
IES(K)= IES(IEXPIV)
IOS(IDXPIV)= J
1844 .
                      14
1845.
1846 .
1047.
                                          KCFT= -KCET
```

```
. .44.
                    15
149.
inbl.
 152 .
. ....
 1534 .
14,00
 1 7 .
                                CUNTINUE
                    16 CUNTINUE
1458.
                    KP= IPS(N)
IF (UL(KP.N)) 19.19.19
14 CALL SING(2)
ISING= 1
1499.
1460 .
1851 .
1462 .
                     19 RETURN
1 454 .
                          END
1305.
               C
                         SUBROUTINE SOLVE (NN.UL.B.X)
IMPLICIT REAL *8(A-H.O-Z)
COMMONZINIZ IPS(30).MDET.KOUNT.ISING
DIMENSION UL(10.10).B(10).X(10)
1-7:
                         N= NN
1.73.
                         NP1 = N+ 1
IP= IPS(1)
                         X(1)= E(1P)
DO 2 I= 2.N
IP= 195(I)
13/6.
                      IM1 = I-1

SUM = 0.0

BC 1 J = 1.[M1

1 SUM = SUM + UL(IP.J)*X(J)

2 X(I) = E(IP) - SUM
1 .74.
 A 74.
lant.
1.42.
                   - . . .
1 . 4 . .
 , ....
1-70.
 . .4 .
 ...
                      4 X(1)= (X(1)-SUM)/DIV
HETURN
END
 . ...
. ::::
                    SUBROUTINE SING(INHY)

11 FORMAT(IX. MATRIX WITH ZERO ROW IN DECOMPOSE.")

12 FORMAT(IX. SINGULAR MATRIX IN DECOMPOSE. ZERO DIVIDE IN SOLVE")

21 IF (IWHY- 1) 1.1.2

1 WRITE (6.11)
```

```
1908 .
                                                                        GO TO 10
WRITE (0.12)
1409.
 1910.
                                                             10 KETURN
                                                                            END
 1411.
                                                                            SUBROUTINE CERUG (NODE)
IMPLICIT REAL*8 (A-H,0-Z)
1912.
 1913.
                                                                           REAL #4 HIN
 1914 .
 1915.
1910.
                                                                            INTEGER PD.PDI.PD2.SS.RP.ZFLAG.RS.P.DD.DD1.HL1.AL2
INTEGER 2 JH(350).DIGNA(952).KINEAS(1302).IDBAS(1302)
INTEGER 2 ISTYPE.LA.LE.1A.IE.PUN.LC(20).IC(8C2)
DOUBLE DASCISION F(8000)
1918.
                                                                              MURLE
 1 119.
                                                                                                       A(4000).C(800).CMIN.COND.ERMAX.SUMINF
                                                           CUMMON DSUM, DPROD.DY, DE, DP, B(350).X(350).Y(350).YTEMP(350).

1A.E.CMIN, COND.FRNAX.SUMINF.ICNAM(1302.2).NAME(20).

2NTEMP(2P).KINP.ITIN.JT(M.ITINV.JTINV.MSTAT.IOBJ.TROWP.IVIN.IVOUT.

3ITCNT.INVFRO.ITPLIM.IFFEZ.JCCLP.NRCW.NCOL.NELEM.NETA.NLELEM.NLETA.

4NGELEM.NINF.NUEL.FN.NUETA.NNEGDJ.NLINES.ISTYPE(350).

5LA(1302).LE(2002).PUN(8).

6IPUNC.NUECI.NDUAL.NIPIW.IFEAS.IFCRSH

CCHMCN 1TCH.ITCHA.IFPIWT.IFNEG.KGUTB

CLMMCN 1A(4600).IE(8000)

COMMCNZELESIZZDI(10.10).BF2(10).EI(10).E2(10).C.IC.LC

COMMCNZELESIZZDI(10.10).C2(9.10).FI(9.10).F2(9.1))

COMMCNZELESIZZDI(10.10).G2(400.10).FA(350).EB(400)

COMMCNZINGXIZ NUF(10).MUF(10).NU(10).MUF(10)

COMMCNZINGXIZ NUF(10).MUF(10).NUF(10).MUF(10)

COMMCNZINGXIZ NUF(10).KINBAS.IDBAS

COMMCNZINGXIZ NUF(10).MUF(10).NUF(10).MUF(10)

COMMCNZINGXIZ NUF(10).KINBAS.IDBAS

COMMCNZINGXIZ NUF(10).MUF(10).NUF(10).MUF(10)

COMMCNZINGXIZ NUF(10).KINBAS.IDBAS

COMMCNZINGXIZ NUF(10).MUF(10).NUF(10).NUF(10)

COMMCNZINGXIZ NUF(10).KINBAS.IDBAS

COMMCNZINGXIZ NUF(10).K
 1921.
                                         C
1422 .
 1924 .
1425.
1426.
 1928 .
 1929.
 1930 .
1931 .
1932 .
 1433.
1434 .
 1430.
1936 .
 1438.
 1439 .
1940 .
1441 .
 1942 .
 1443.
                                                                           RETURN
1944 .
                                                             3) WRITE (5.40)
WRITE (6.41)
WRITE (6.49)
                                                                                                                                           (BA(1),1= 1.M)
 1945.
                                                                                                                                      (8E(I), I=1.K)
GO TO 35
1946 .
 1947.
                                                                                        (KNU. EG. 0)
                                                                           WRITE (6.42) ((G1(I.J).1= 1.M).J= 1.KNU)
IF (KNU.EG.O) RETURN
WRITE (5.43) ((G2(I.J).1= 1.N).J= 1.KMU)
 1944.
1449 .
1950 .
1951 .
                                                                             RETURN
                                                            4) FORMAT (/.1x. LINEAR CONSTRAINTS.)
42 FORMAT (1X. G1= '.7F12.5)
41 FORMAT (1X. BA= '.7F13.5)
49 FORMAT (1X. BB= '.7F13.5)
 1452.
 1453.
1954 .
1455.
                                                           49 FORMAT (1x, BH= ',7F13.5)
43 FORMAT (1x, G2= ',7F13.5)
60 IF (MODE - 4) 65,70.100
60 WRITE (6,85) (KINBAS(1).1=1,NCOL)
WRITE (6,85) (JH(1).1= 1,NFCW)
80 FORMAT (1x, KINEAS= ',2014)
85 FORMAT (1x, JH= ',2014)
 1956 .
 1457.
 1438.
1959.
 1 461 .
1402 .
                                                                           RETURN
                                                            70 WRITE (6.71) KNU, KNU
WRITE (6.152) (NUH(I), I=1.IH)
WRITE (6.151) (MUH(I), I=1.IH)
WRITE (6.74) (XX(I), I=1.IH)
WRITE (6.75) (PI(I), I=1.IH)
 1463.
1404 .
 1965 .
 1466 .
1967 .
```

```
71 FORMAT (//.' SUPEREASIC INFC: KNU=',13.' KMU= ',13)
74 FORMAT (' T =',9F12.E)
75 FORMAT (' LAMBDA=',9F12.E)
1968.
1969 .
1970.
 1971 .
                                                      RETURN
                                        100 IF (MODE - 6) 101.140.
101 WRITE (6.120) KNU.KMU
IF (KMU.FG.C) GO TO 105
                                                                                                        101.140.140
1972.
1 +73 .
1474 .
                                                      IF (KMU.FG.0) GBF1(I).I=1,KMU)
WRITE (6.121) (BF1(I).I=1,KMU)
WRITE (6.122) ((D1(I.J),J=1,KMU),I=1,KMU)
WRITE (6.123) (E1(I).I=1,KMU)
TE (KNU.EG.0) GD TO 105
TE (KNU.EG.0) GD TO 105
1975.
 1976.
1977.
                                        WRITE (6.123) (E1(I).I=1,KML)
IF (KNU.EG.0) GO TO 105
WRITE (6.124) ((F1(I.J).J=1,KNU).I=1.KMU)
105 IF (KNU.EG.0) RETURN
WRITE (6.125) (BE2(I).I=1.KNU)
WRITE (6.126) (E2(I).I=1.KNU)
IF (KMU.EC.0) RETURN
WRITE (6.127) ((C2(I.J).J=1,KMU).I=1.KNU)
WRITE (6.128) ((F2(I.J).J=1,KMU).I=1.KNU)
RETURN
1978 .
 1474.
1980.
1481.
1982 .
1483.
 1984 .
1485.
                                                      RETURN
1986.
                                      RETURN
120 FORMAT (1X.*BL CCNSTRAINIS. KNU.KMU:*.213)
121 FORMAT (1X.* BF1= *. F14.6)
122 FORMAT (1X.* D1= *.F14.6)
123 FORMAT (1X.* E1= *.F14.6)
124 FORMAT (1X.* E1= *.F14.6)
125 FORMAT (1X.* BF2=*.F14.6)
126 FORMAT (1X.* BE2=*.F14.6)
127 FORMAT (1X.* E2= *.F14.6)
128 FORMAT (1X.* F2= *.F14.6)
129 FORMAT (1X.* F2= *.F14.6)
140 WRITE (6.150) PD.KMU.KNU.TNEQ
WRITE (6.151) (MUH(I).I= 1.IH)
WRITE (6.152) (NUH(I).I= 1.IH)
150 FORMAT (//.1X.* ENDPCINT FINDER DATA:PD.KMU.KNU.INEQ= *.413)
151 FORMAT (1X.*NUH(I)= *.2013)
WRITE (6.154) (JH(I).I=1.M)
WRITE (6.154) (JH(I).I=1.M)
WRITE (6.155) (DIGMA(I).I=1.N)
154 FORMAT (1X.*UHCA= *.1514)
WRITE (6.153) (XX(I).I=1.NM)
WRITE (6.156) (PI(I),I=1.NM)
WRITE (6.156) (PI(I),I=1.NM)
157 FORMAT (1X.* XX= *.9F13.6)
158 FORMAT (1X.* XX= *.9F13.6)
RETURN
END
                                        120 FURMAT
                                                                             (1X. BL CENSTRAINTS.
1987.
                                                                                                                                                         KNU.KMU: . . 2131
1988.
1939.
1990 .
1491.
1992 .
 1993.
1994 .
1495.
1996 .
1997.
1998.
1999.
2000.
2001.
2002.
2003.
2004.
2005.
2005.
2007.
2008.
2009.
                                                      RETURN
2010.
2011 .
                                                      END
```

III. The HRA (Homotopy Retraction Algorithm) Code for Solving Equilibrium Problems

III.1. Revisions to the Original Code

The homotopy retraction algorithm is described in detail in Chapter IV of the report [2]. However, since the completion of that work the author has changed the HRA code to improve the domain of convergence of the code with respect to the user supplied initial utility levels. With these changes the code successfully finds equilibrium points for problems and starting points which resulted in failure for the previous version of the HRA. Also, solution times have been as fast, or faster, than before. This version of the code has not been tested on a sufficient number of test problems but the advantages over the old code seem to be so great that we should describe the code in its present form.

By Lemma II.2.4, solving the equilibrium problem is equivalent to solving for

$$w \equiv (\pi, \lambda, \zeta^{i}(i \in m), p; s, t, z^{i}(i \in m), p)$$

which satisfies

$$f(w) = 0, \qquad w \in D, \qquad (5)$$

where

$$f(w) = \begin{pmatrix} \pi(w)w^{1} - \lambda_{1}(w)(t_{1}(w) + v_{1}) \\ \vdots \\ \pi(w)w^{m} - \lambda_{m}(w)(t_{m}(w) + v_{m}) \end{pmatrix}$$

 $(\pi(w))$ refers to the first m components of w, etc., and

$$D \equiv \{w | \gamma^{i} z^{i} - t_{i} = v_{i} \qquad i \in \underline{m}$$

$$\sum_{i=1}^{m} B^{i} z^{i} + pe + s = \sum_{i=1}^{m} w^{i}$$

$$\pi B^{i} - \lambda_{i} \gamma^{i} - \zeta^{i} = 0 , \qquad i \in \underline{m}$$

$$\pi e - p = 1$$

$$\pi s = 0$$

$$\zeta^{i} z^{i} = 0 , \qquad i \in \underline{m}$$

$$w > 0 \qquad \} .$$

The original homotopy which is used to define a path to the solution of (1.2) is

$$F(w, \theta) = \theta f(w) - (1 - \theta)(\lambda(w) - \lambda^{0})$$

where λ^0 is determined by the solution to the auxilliary linear program (2). As part of the modification to HRA we changed the deformation above to

$$F(w, \theta) = \theta f(w) - (1 - \theta)(t(w) - t^{0})$$
.

Here t(w) refers to the slack variables of the first m rows, where m is the number of consumers in the economy. By the properties of the auxiliary linear program, it will always be true that t^0 , the value of t(w) at the optimum, is identically zero. This is because one can never achieve a higher level of exports p by allowing consumers to have a greater utility level than v_i . Thus, we can simplify the definition of the homotopy to $F:DX[0,1] \to R^m$,

$$F(w, \theta) = \theta f(w) - (1 - \theta) t(w) . \tag{6}$$

A convergence theorem using the path defined by (6) can be proved even though we will not do so here.

Other amendments to the algorithm include a procedure for altering the initial utility levels v_i , $i=1,\ldots,m$, if the processing of the auxiliary linear program indicates that the solution is unsatisfactory. There are three factors which can cause the program to change the initial utility levels:

- 1. If the l.p. is infeasible, then $\,\mathbf{v}_{i}\,$ is too large. They will all be reduced.
- 2. If some $\lambda_{\mathbf{i}}^0 = 0$ initially, it indicates that $\mathbf{v_i}$ can be increased without changing the optimal value of the objective function. For each i for which this is true we increase $\mathbf{v_i}$ until $\lambda_{\mathbf{i}} > 0$.

3. If $f_i(w) \leq 0$ for some i, the initial boundary conditions are not satisfied. By reducing v_i for such indices i, we can increase f_i until it is positive.

When we say to increase v_i or decrease v_i , we mean that the following procedure is followed:

- a) After each solution of the auxiliary LP, a scalar TI is incremented by one starting with TI = 5.
- b) If any of the three conditions above occurs, to reduce $\mathbf{v}_{\mathbf{i}}$ we let

$$v_i = v_i * (1 - 1/TI)$$
,

to increase v_i we let

$$v_i = v_i * (1 + 1/TI)$$
.

c) When TI reaches 20 and the solution to the LP is still unsatisfactory, the program terminates.

III.2. Input Requirements

The form of the input data for the HRA is exactly identical to the form for the BCA. See Section II.1 for a description.

III.3. Main Program

The main program consists of 390 source statements which perform many of the same functions as the main program for the BCA. The parameters are initialized and possibly altered by reading the namelist PARM1. The subroutines of LPM1 are called to input and solve the auxiliary linear program. Tests 1 and 2 of Section 1 are performed. The data is read so the C matrix can be constructed. Then the budget surpluses \mathbf{f}_i are calculated and test 3 is performed. The rest of the main program is essentially identical to that of the BCA except that different decisions are made after the ENDPNT subroutine has been called to determine which is the next cell (see Figure IV.3.1, Elken [1977b]).

Restrictions Relevant to the Use of HRA

- 1. The number of consumers (IH) must be less than or equal to 10.
- The matrix D must not have more than 350 rows or 400 columns or more than 4000 non-zero elements.

Of course, if one must solve a problem larger than these dimensions allow, more core must be allocated and the dimensions of the appropriate variables must be changed in every subroutine in which they appear.

Before describing the other subroutines of HRA we will define the variables which differ from those in BCA as described in Section II.2.

Variable Glossary

In HRA we refer to one more variable from the blank common of LPMI.

We use MSTAT to check if the current auxiliary linear program is infeasible.

The common blocks LF1, BLCST, BLCST2, LNCONS, INDX1, and INDX2 are identical in both HRA and BCA. COMMON/INT/ contains the variable KEND, the number of times the ENDPNT subroutine is called.

We include this variable to pass its value from the main program to ENDPNT so that we can test to see whether this is the first call to ENDPNT or not. In addition to those variables described in Section II.2, COMMON/SCAL/ contains the variable

IHP1 = IH + 1 the number of consumers plus one. This is the dimension of the consumer space plus one for the homotopy parameter.

COMMON/DIM/

The description of the algorithm in Chapter IV of Elken [2], shows when ITAIL is set equal to 1.

The rest of the common blocks are identical to those in BCA.

III.4. Subroutines of HRA

We will describe only the differences between the subroutines of HRA and those of BCA.

1) Subroutine BLCONS: Calculates the coefficients of the bilinear functionals the budget surpluses so that f(w), (6), can be evaluated in terms of the superbasic variables (see VI (4, 9) of Elken [2]. This subroutine is substantially the same as BLCONS in BCA except that in this case all the bilinear functionals are computed rather than the first DD, the number of bilinear constraints which are currently satisfied.

Subroutines called: UPAKC

- 2) SUBROUTINE FINDP (PD1, IS, P): same as in BCA. See Section II.3.
- 3) SUBROUTINE PIVOT (SS, RR), SUBROUTINE UPAKC (II), SUBROUTINE

 BSCNG (S, R), SUBROUTINE SUPERB (KEY, PK1, IS, (PD2, JS), and

 SUBROUTINE RECALC: same as in BCA, see Section II.3.
- 4) SUBROUTINE ENDPNT (JS, PD1, IS, NET): This routine implements the path-following algorithm described in Section IV.2 of [2]. The reader should consult that work to be able to understand this subroutine. Here we will describe some details which were not mentioned in Section II.3.

The independent variables (superbasics) are placed in a vector X of dimension IH + 1 = IHP1. The extra dimension is for the homotopy parameter, θ . Theta always occupies the last position in X, X(IHP1).

From computational experience, we know that the curve $F^{-1}(0)$ is highly nonlinear in the last variable when the algorithm begins. That is, for the first tangential approximation in the first cell, θ increases very quickly, initially, but quite slowly thereafter. Remember, θ increases from 0 to 1 as the algorithm progresses, although some decreases are possible. To remedy the poor guess at θ which would result from following the initial tangent all the way to the opposite boundary of the cell, on the first iteration we move only one quarter of the distance to the opposite boundary. This rather ad hoc procedure seems to work quite well. After this initial step, the algorithm described in the work cited is followed precisely. The first tangent of the first cell is characterized by the fact that KEND = 1 (KEND is the number of cells traversed) and INFL = 0 (INFL is set equal to 1 after the first steplength is reduced).

In this program, the integer variable MFLAG has a different meaning than in the BCA code

when the tangent to the curve is being computed,

when a "hyperplane subproblem" is being solved,

when an endpoint in the opposite facet is being

2 when an endpoint in the opposite facet is being solved for, and
3 when the equilibrium point appears to be in the current cell. Newton's method will be implemented as a tail routine on F(X) = 0.

MFLAG =

SUBROUTINES CALLED:

QUADS, DSENT, CONCHK, GTN, FTN, DERIVG, DERIV, NORM, DECOMP, SOLVE.

Subroutines QUADS (U, IMAG, ALPHA, BETA) and DSENT (ALPHA, PNORM) are the same as in BCA.

SUBROUTINE GTN (IFL, Q, U, F) evaluates the homotopy $F(w, \theta)$. When one writes this function in terms of the superbasic variables one has

$$THETA = X(IHP1)$$

$$Q(X) = THETA*f(X)$$

$$-(1 - THETA)* \begin{pmatrix} KNU \\ \sum_{I=1}^{KNU} G1(MUH(J), I)*X(KMU + I) + BA(MUH(J)), & J = 1,...,KMU \\ X(KMU + J), & J = 1,...,KNU \end{pmatrix}$$

The values of the variables of t indexed by MU (the first KMU in the vector above) are saved in the vector BLAM(10). These values are used in the subroutine DERIVG to be described later.

The last component of Q contains the value of a functional determined by the value of IFL (MFLAG in subroutine ENDPNT). If IFL = 1, then

Q(IHP1) =
$$\sum_{I=1}^{IH} ACC(2, I)*(X(I) - ACC(1, I))$$
.

If IFL = 2, then

$$Q(IHP1) = \begin{cases} \sum_{I=1}^{KNU} G1(JJ, I)*X(KMU + I) + BA(JJ), & \text{if MPD} = 1, \\ \\ KMU \\ \sum_{I=1}^{KMU} G2(JJ, I)*X(I) + BB(JJ), & \text{if MPD} = -1. \end{cases}$$

The subroutine FTN is called to evaluate f(X).

SUBROUTINE DERIVG (IFL, U, G, F): calculates the jacobian of the function evaluated by GTN. The jacobian is of the form

$$DG(X) = \left[[THETA*Df(X) - (1. - THETA) \left(\begin{array}{c|c} 0 & Gl_{MUH}, \\ \hline 0 & I \end{array} \right) \right] f(X) + \left(\begin{array}{c|c} BLAM \\ \hline X_{NU} \end{array} \right) \right]$$

SUBROUTINE CONCHK (GMIN, GKMIN, MP2): this subroutine evaluates the basic variables in both the primal and dual systems, saves the value of the minimum in GMIN and points to which variable it is with (KGMIN, MP2). From the theory of the homotopy retraction algorithm, it is known that the first IH primal and dual variables cannot become negative so we ignore these variables when searching for the minimum. Also, the primal and dual variables corresponding to the objective function are of no importance to the algorithm, so we do not compare these values when searching for the minimum.

SUBROUTINE FTN(F, Y): evaluates the bilinear budget surpluses and stores them in F. This subroutine is identical to that in the BCA code except that no additional functionals are evaluated. That task is performed by GTN as described above.

SUBROUTINE DERIV: evaluates the jacobian of the function described in FTN. This is identical to the subroutine by the same name in the BCA code except, again, it is simpler because no rows need be appended because of an additional functional.

SUBROUTINES NORM (Y, S1, NY, DECOMP (NN, A, UL), SOLVE (NN, UL, B, X), SING (IWHY), DEBUG (MODE): are identical to the subroutines with the same names in BCA.

III.5. Sample Problems

Below we give the output of the HRA program for the same two problems (MAS-Colell and Whisman) presented in Section II.4. The form of the input is identical for HRA and BCA so we will not repeat the input decks here.

The output is very similar. The reader will note that the subproblem solved by ENDPNT is (IH + 1)-dimensional with HRA while the dimension grew from 1 to IH for the BCA. On these two problems, though, the HRA code required fewer calls to ENDPNT before equilibrium is reached. This behavior is typical with larger problems as well.

	E0= ,9009				3-11-6 = 949999999999999999999999999999999999		60666	1:	*AX=	.100		NELEM 122
7.SE.C	70-05.TOL			- NPMAN.	Z		606666666666		350 .NT MAX=	45= TOLR I=		Z 4 4 A
CPU 0MIN 00.17SEC	TCLBD= .10060000000000000000000000000000000000	100		E= -1.	99999, ZTOLRP= .							-1.00000000
STOP 77201.1245 CPU	3.TOLCV= .9999	O.ITLIME=		30035E-03.ZSC	30, ITRLIM=							NNE GOJ
	BD= .10060000000000000010-03.	0.1		.dost= .10000	NOUAL =	.10000000E-08						VECOUT 60001 60001
RT 77201.124	.1000000000 PMX= 10.000	C,KOUTB=		0034E-05,ZTC	I = 1.	0.ZTOLDA= .1						VECIN 60002 60002
16F3751 JCH /TREWEGGE/ START 77201.1245	000-09,TCLB0=	2.IFRUD=	1		6.ITCH=				948434 SEC.	IN EASIS	0.63725E-16	CBJ VALUE 0.22100002 -).61999995
F3751 JCB	.1c0cc0000c00000000-c9.		CHC 0 3	.10070005E-03,ZIOLPV=	8000.IALG= 08J=	.100000043E-11,KINB=	70	M STATISTICS ROWS STPUCTUPAL COLUMNS STPUCTUPAL COLUMNS	IS NOW 14.	STRUCTURAL COLUMNS IN STRUCTURAL COLUMNS IN VECTORS ABOVE BUMP VECTORS HELOW BUMP 9 NONZ 3 ETAS	S NON	N N N N N N N N N N N N N N N N N N N
	550-00.	3,L=	EEND IECHO, IH, I CECHC EPARAM	110	:	27ETA= .10000 EEND	PROBLEM MAS-COL	OPROPLEM STATISTICS 6 ROWS 6 STRUCTUPAL C 12 NON-ZERO ELE	THE TIME LEFT IS NOW 14,648434 INVERT STATISTICS	3 VECTORS H	U: 3 NONZ 1 ETA TOTALS: 8 OFF DIAG PELATIVE EPRCS IN X =	ITCOUNT STATUS 1 0 MAS-COL
	S	Ï,				0		90				

```
0.0000000
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0.0
                 DI(I)
3.53000000
0.25000000
1.00000000
0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          0.03394 0.58200
-0.002244 -0.000898
0.03394 0.61222
-0.000000 0.0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       THE CURRENT GALL:

THE CURRENT GUAGRATIC APPROXIMATION TO THE CURVE IS:

6.5550000 0.9718479 0.0

6.0200002 0.0741745 0.0

6.0200002 0.02203737

WE MOVE A DISTANCE 0.420119 ALGNG THE APPROX.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    0.5000000
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      15:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        TO THE CUPVE
               ROW NAMES
UTIL1
UTIL2
UTIL3
GOOD1
GOOD2
UBJ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              0.024685
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           O.23181
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         0.02000
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                CLURRENT GUAGGATIC APPROXIMATION TO APPR
                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                                                           SMAL= 0.9090909 THETA= 0.4761905
WE RETRACT CNTC THE 1 TH EOUNDARY
XO= 0.000000 0.554545 0.231
INITIAL BUDGET SURPLUSES: 0.02000
                                                                                                                                                                                                                        OF WATRIX.
                                                                                                                            SE
              VALUE

(**93000000

0.*92000000

3.*92000000

0.*82100000

0.*82000000

1.*906462 SI
                                                                                                                                                                                                                                                                                                                 00000
                                                                                                                                                                                                                       STRUCTURAL COLUMNS
                                                                                                                                                                                                                                                                                                   COLUMNS 1 THEUGH
                                                                                                                           FT 18
                                                                                                                           1
              JH(I)
ACTI
ACT2
ACT3
GG002
MEXP
0BJ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          THE
```

```
PI(I)
0.63397455
0.63367455
0.23660254
0.73205090
0.26794910
                                                                                                                                                                                                                                           C X= 0.60179 0.21281 0.26594 1 X= 0.063386 0.204450 -0.003978 1 X= 0.63386 0.20485 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.30576 0.20470 0.30550 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.30500 0.3
0.0339351
0.6122210
ALONG THE APPEOX
NUMBER 5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          1.1547035
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      POW NAMES
UTIL1
UTIL2
UTIL3
GOODI
GRODD
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        ENDPNT CALLS, WE PAVE ECUILIBRIUM
       0.5916551
0.5916551
0.394011 A
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      SEC.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   THE TIME LEFT IS NOW 14.876805 SI
UTILITY LEVELS: 1.5773504
PRICES: 0.633975 0.866025
MAS-COL
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          VALUE
1.57735641
1.15473049
4.22649816
6.630000023
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     SCALAR FUNCTION CALLS=
JACOBIAN EVALUATIONS=
       C.0115164
O.0
MOVE A DISTANCE
CURVE HIT A 1 TYPE
                                                                                                                                                                                                      MFLAG= 2

ITERATION:

ITERATION:

NORM(F(X))=

ITERATION:

NORM(F(X))=

ITERATION:

NORM(F(X))=

AFTER 3 NEW
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               2
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         TOTALS:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             ACTI
ACTI
ACTZ
ACTZ
ACT3
UTILI
MEXP
0BJ
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               AFTER
       1598

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

16999

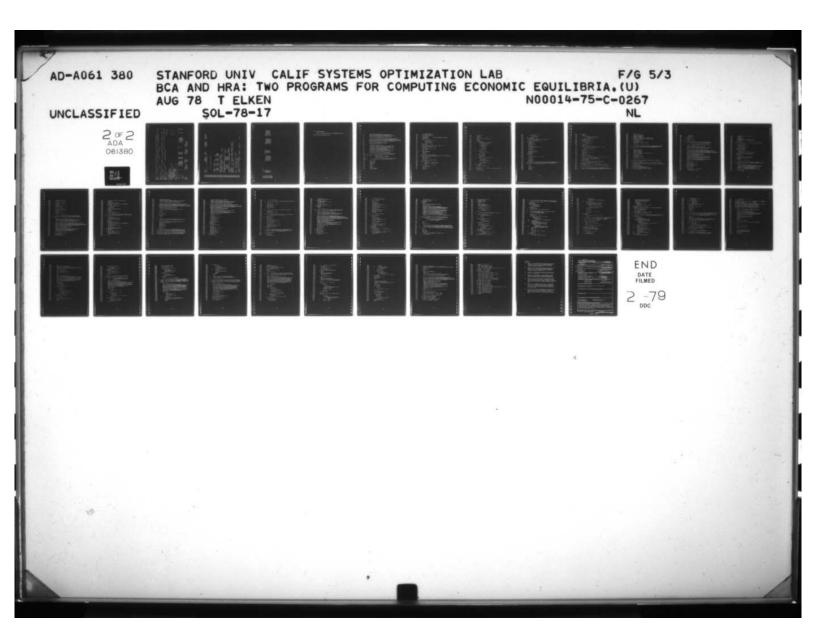
16999

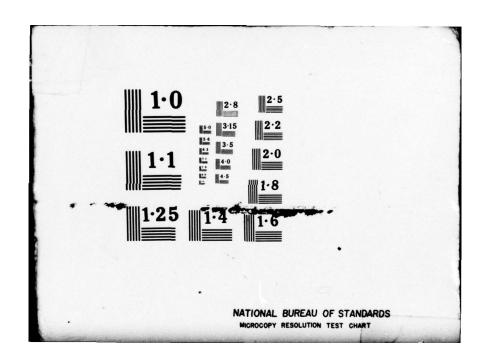
16999

16999

16999

169
```





		. NP MAX=	5. IFP	.999999755-05.2	6600666666666	3	S= S=	TOL.P.1= .100	NFTA NFLFM 65 22 7 226 8 331 10 42	
U DMIN DO.19SEC	99999999999999999999999999999999999999	E= -1.00000000	= dN 1X * 06006	99999, ZTGLRP= .9	ŏ		AS=	D.	1.000000000000000000000000000000000000	2000
125	104	.100000005E-03.ZSCALE=	=MIDIN.000	30,1TPLIM=					N N O O 0 0 0 0 0 0 0 0	200 - 0.48900000000000000000000000000000000000
1259 STOP	0000000000		I.NDUAL=	FPQ= 3					VECOUT MEXP 600D2 600D1 C2AC1 C3AC4 C3AC4	0.0000000000000000000000000000000000000
A	5	.100000034E-05.ZTC0ST=	=193	30.1NVF PQ=					VECIN 39 C3AC4 39 C3AC4 96 C2AC2 96 C2AC2 92 C4AC2	NAMES UTILI UTILI
JCB ZTREWB668Z ST	005-09.1CLE		C.NDEGI	E,IICH=			6	AFP FASTS NONZ & ETAS	CPJ VALUE 9.95999039 1.38999939 -0.38874996 -0.5649996 -2.57999952 -2.57999992	VALUE C.499000000 C.32000000 C.32000000
ICF3751 JC	1. IF C F C = 3	H.ICECHO 0 4 1	8C39. [ALG=	1.108J= 8 100001043E-11.KINB=	WHISMAN	ROWS STRUCTURAL COLUMNS STRUCTURAL COLUMNS NON-ZERO ELEMENTS	INVERT STATISTICS 20 NONZ IN BASIS	TOKAL CCCCMNS AS AECCE BUMP AS BELCW BUMP NZ 2 ETAS NZ 3 ETAS 12 OFF DIAG NG	2 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	> >
SPARM1		EPARAM ZTOLZE .1000000	NEMAX=		PRCBLEM WH	PROBLEM STA	INVERT STAT	S STRUCTOR 2 VECTORS 6 VECTORS 10 9 NONZ 10 1 9 NONZ 10 1 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2	11CCUNT S1	0.14C1)

```
5.030000000
111.33333300
9.00000000
                                                                                                                                                                                                                                    1.000000
                                                                                                                                                                               0.500000000
-7.000000000
-1.000000000
                                                                                                                                                                                                                                    1.0000000
                                                                                                                                 0.941406
                                                                                                                      0.0000000
                                                                                                                                                                                                                                    0.500000
                                                                                                                                                                                                             I ENDPNT CALLS. WE HAVE EGUILIBRIUM.
GC001
GC0003
G00003
08J
                                                                   00000
                                                                                                      SMAL= 1.1718749 THETA= 0.5395683
WE RETRACT CNIC THE 1 TH BOUNCARY.

XO= 0.003300 1.693750 0.441406

INITIAL BUDGET SURPLUSES: 0.00000
                                                                                                                                                                                                                    97
                                                                                                                      0.00030.0
                                                                                                                                                                                                                                THE TIME LEFT IS NOW 14.865749 SEC-
UTILITY LEVELS: 1.3333334
PRICES: 1.50000 2.50000
Z.58033300
C4AC2
GUGD3
GUGD3
OBJ
THE TIME LEFT IS NOW 14.886756 SEC.
                                        STRUCTURAL COLUMNS OF MATRIX.
                                                                                                                                                                                                                    SCALAR FUNCTION CALLS=
                                                                400mc=n40
                                                               MFLAG= 2
NUMM(F(X))=
1 TERATION :
NORM(F(X))=
AFTEH I NE.
                                                                       00000
                                                                COLUMNS
-0.4803
                                                                                                                                                                                                                    FUTALS:
```

ROW NAMES UTIL2 UTIL2 UTIL4 50001 60003 60003 VALUE C.80000001 C.80000001 C.7333314 -0.2066687 3.40000015 WHISMAN C1AC1 C2AC2 C3AC1 C4AC1 WEXP C6AC2 G0003

III.6. HRA Source Listing

Below is a listing of HRA without the subroutines which comprise the linear programming code LPM1.

```
IMPLICIT REAL*B (4-H.O-Z)
REAL*A ZTOLZE.ZTOLPV.ZSCALE.ZTETA.ZTOLRP.ZTOLRI
INTEGER PD.PO1.PD2.S.R.SS.RR.ZFLAG.RS.P.QN
INTEGER*2 JH(350).DIGM.4(952).KINBAS(1302).IDBAS(1302)
INTEGER*2 ISTYPE.LA.LE.IA.IE.PLN.LC(20).IC(800)
DDUGLE PRECISION E(8000)
REAL A(4000).C(800).CMIN.COND.ERMAX.SUMINE
     5.
     5.
                                  C
                                                       COMMON DSUM.DPROD, DY.DE.DP.B(350).X(350).Y(350).YTEMP(350),
1A.S.CMIN.COND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).
2NTSMP(2C).KINP.ITIM.JTIM.ITINV.JTINV.MSTAT.IDBJ.IROWP.IVIN.IVOUT.
3ITCNT.INVSSO.ITRLIN.IFFEZ.JCOLP.NRCW.NCJL.NELEM.NETA.NLELEM.NLETA.
4NGFLCM.NINF.NUSLEM.NUETA.NNEGDJ.NLINES.ISTYPE(350).
5LA(1302).LE(2002).PUN(8).
6IPUNC.MDEGI.NDUAL.NIPIW.IFEAS.IFCRSH
COMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTE
COMMON IA(4000).IE(8000)
10.11.12.13.
 14.
 16.
17.
17.1
                                                      DATA QN/'N '/
COMMON/LP1/PI(1302).XX(1302)
COMMON/LP1/PI(1302).XX(1302)
COMMON/RLCST/9F1(10).3F2(10).E1(10).E2(10).F2(9.10)
COMMON/RLCST/D1(10.10).D2(9.10).F1(9.10).F2(9.10)
COMMON/LNCNS/G1(350.10).G2(40C.1C).BA(350).EB(400)
COMMON/INDX1/ NUH(10).MUH(10).NU(1C).MU(10)
COMMON/INDX2/ JH.DIGMA.KINEAS.IDBAS
COMMON/INDX2/ JH.DIGMA.KINEAS.IDBAS
COMMON/SCAL/ 9T.NB.JJ.MFLAG.IHP1.P.PD.MPD.KFUN.KJAC
COMMON/INT/ IPS(30).KDET.KEUNT.JISING.KEND
COMMON/INT/ IPS(30).KDET.KEUNT.JISING.KEND
COMMON/INT/ IPS(30).KDET.KEUNT.JISING.KEND
COMMON/INT/ IPS(30).KDET.KEUNT.JISING.KEND
COMMON/INT/ IPS(30).KDET.KEUNT.JISING.KEND
COMMON/INT/ IPS(30).KDET.KEUNT.JISING.KEND
NAMELIST/PARMI/TOLF//TOLBD.TOLCV/THETA.STPMX.STPRD
NAMELIST/PARMI/TOLF//TOLBD.TOLCV/ICNTRL.IECHO.
1STPMX,STPRD.ICECHO.IH.L.IPFGD.KOUTE.ITLIME
                                  C
 17.2
 14.
20.
 23.
25.
 214 .
 24.
30.
31.
32.1
32.2
32.3
32.4
32.6
32.6
7
                                               HRA: A CODE WHICH IMPLEMENTS THE HOMOTOPY RETRACTION ALGORITHM FOR SOLVING ECONOMIC EQUILIBRIUM PROBLEMS.
                                 ουυυυσίτοι
                                                                                                       THOMAS P. ELKEN
SYSTEMS OPTIMIZATION LABORATORY
OPERATIONS RESEARCH DEPARTMENT
                                                           AUTHOR:
 32.8
32.9
32.91
32.92
32.93
                                                                                                        STANFORD UNIVERSITY
                                                       FOR DESCRIPTION OF THE ALGORITHM AND DOCUMENTATION OF THIS PROGRAM SEE SOL TECHNICAL REPORT 77-26.
 ₹3.
                                                             TOL30 = 10-04
24 .
                                                            TOLE Z= 10-10
STOMX= 10-0
  37.
                                                            STORD= 0.5

ITLIME= 100

ICNTRL= 1

ISCHO= 0
 38.
  30.
 40 .
4 ? .
                                                            KOUTHE C
                                                            ICECHO=
 43.
45.
                                                            KJAC= 0
IPROD= 1
46.
                                                            WRITE (6.PARMI)
```

```
IF (!CNTRL) 10.40.40
10 CALL INPUT(!ACT)
IF (!ACT) 30. 20. 20
20 IF (!ECHO.FG.0) GO TO 29
51.
                           20 IF (IECHO.FO.0

MUB= NROW

WRITE (6.21)

21 FORMAT (////,*

22 L3= MUB+1

MUB= LB+ 14

K= 0
54 .
55.
                                                                                       STRUCTURAL COLUMNS OF MATRIX. 1)
50.
                          K= C
DD 25 J= L8.MUR
IF (J.GT.NCOL) GO TO 26
K= K+ !
CALL UNPACK(J)
DD 24 I= 1.NROW
G1(I.K)= Y(I)

24 CONTINUE
25 CONTINUE
26 WRITE (6.27) L8.MUR
27 FDEMAT (////, COLUMNS '.I4.' THRUOGH '.I4)
DO 23 I=1.NROW
WRITE (6.28) (G1(I.J).J=1.K)
28 FORMAT (IX.15F8.3 )
23 CONTINUE
60.
61.
63.
64 ·
71.
72.
73.
74.
                           23 CONTINUE

IF (MUB .LT. NCOL) GD TC 22

29 CALL NORMAL

CALL UNFAVE(6)

GO TO 10
75
76
77
77.1
                   77.2
77.3
77.4
76.2
78.3
76.4
78.6
78.7
78.6
78.9
79.1
70.2
79.4
79.5
79.6
79.7
79.8
79.9
80.1
80.1
90.3
                   C
                                  CALL DEPUG(1)
CALL SHIFTP(3,4)
KEND= C
KENDSV= O
IHL= IH+ L
86.
87.
98.
94.
oc.
                                   M = NECW
91 .
```

```
N= NCOL- NEOW

NM= N+ M

L1= M- L

MM1= M- I

175(Ny= 1)

15 (1920).50.0) GO TO 43

RFAD (5.1030) (SHR(I).I=1.IH)

43 ICOU=1

13AV= c

LC(I)= 1

DO 68 K= 1.IH

KP1= K+ 1

45 RFAD(KINP.1010) ISHA. LL. KK

16 (1SHR.50.-1) GO TO 50

17 (1SHR.50.-1) GO TO 42

16 (LL.60.0) GO TO 42

17 (LL.60.0) GO TO 47

16 (1COU)= R(L)

GO TO 45

17 (COU= ICOU+ 1

16 (ICOU)= E(I)

CONTINUE

GO TO 45

1 ISAV= ICOU+ 1

1 C(ICOU)= LL

EAD (5.1030) C(ICOU)

GO TO 45

1 ISAV= ICOU+ 1

1 C(ICOU)= L

EAD (5.1030) C(ICOU)

GO TO 45

1 ICOU= ICOU+ 1

1 C(ICOU)= SHR(K)*E(LL)

GO TO 45

1 ICOU= ICOU+ 1

1 C(ICOU)= SHR(K)*E(LL)

GO TO 45

1 ICOU= ICOU+ 1

1 C(ICOU)= SHR(K)*E(LL)

GO TO 45

1 ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICOU= ICOU+ 1

ICONTINUE

GO TO 45

LC(KPI)= ICOU+1

ICONTINUE

GO TO 45

LC(KPI)= ICOU+1

ICONTINUE
       93.
      94.
       97.
        99.
        ga.
 100.
101.
102.
103.
104.
105.
   107.
   109.
 11C.
111.
112.
   113.
 114.
115.
116.
117.
116.
 120 ·
121 ·
122 ·
123 ·
   124.
 125 ·
126 ·
127 ·
  128 .
120.
130.
131.
132.
   134.
135.
136.
137.
136.
                                                                         55 CONTINUE

GO TO 45

58 LC(KP1)= 100U+1

68 CONTINUE

IF (ICECHO.FQ.1) GC TO 69

MUSEC

MRITE (6.61)
 140 .
 141.
  144.
 145.
                                                                         #2[TE (6.51)
61 FORMAT(////...
62 LD= MUS+1
WUR= LP+ TH - 1
                                                                                                                                                                                                                           STRUCTURAL COLUMNS OF MATRIX. .. )
 148 .
  140.
150 .
                                                                                           K= 0
00 55 J= L8.MU8
```

```
IF (J.GT.NCOL) GO TO 66
K= K+ 1
CALL UPAKC(J)
DO 64 I= 1.NROW
G1(I.K) = Y(I)
CONTINUE
152 ·
153 ·
154 ·
155 ·
                        156 .
158.
159.
161.
162.
164 · 165 · 166 · 167 ·
169.
150.
170.
171.
172.
173.
174.
175.
176.
177.
170.
180.
181.
182.
                          BO CONTINUE
PI(J)= DSUM
SO TO 100
90 PI(J)= 0.
100 CONTINUE
K= 0
1PA.
185.
187 .
                                    OG 11C I= 1.NM
IF (KINBAS41) .EQ. 0) GD TC 105
IDBAS(I)=
188.
189.
190 • 191 • 192 • 193 •
                           GO TO 112
                                    DIGMA(K)=
                         OIGMA(K)= 1
38(K)= PI(I)
10345(I)= K
XX(I)= 0.0

110 CONTINUE
THIS IS THE BILINEAR PHASE OF THE ALGORITHM. VARIABLES XX(I).....
XX(M) ARE THE SLACKS. AND PI(I).....PI(M) ARE THE USUAL PI'S.
XX(M+1)....XX(M+N) ARE THE X'S. PI(M+1).....PI(M+N) ARE THE
DUAL SLACKS. MORE INITIALIZATIONS.
194.
108.
193.
200.
                                    LP= 2
                                   LP= 2

ITAIL= 0

PD= 1

JS= 1

DD 120 I=1.1H

MUH(I)= 0

NUH(I)= 0

NU(I)= 0
204.
210.
```

```
212.
213.
214.
215.
216.
217.
                          120 CONTINUE
KNU= 0
KMU= 0
                                   DO 430 I= 1.IH

IF (KINBAS(1).GT.C) GC TO 425

CALL SUPERS(1.1.I.2.0)

GO TO 430
                         CALL SUPERS.

430 CONTINUE
IF (KMU.5Q.2) GD TO 435
DD 433 I= 1.KMU

433 VS(I) = PI(MU(I))

435 IF (KNU.5Q.2) GD TO 440
DD 438 I= 1.KMU
IK= I+ KMU
VS(IK) = XX(NU(I))

635 CALL BLCONS
219.
                                               CALL SUPERB(1.-1.1.C.C)
220 ·
221 ·
222 ·
223 ·
224 .
225.
226.
227.
                          VS(IK)= XX(NU(I))

CALL BLCONS

44C CALL FIN(F.VS)

IO= 1

THFTA= 0.0

VRITE (F.449) (F(I).I=1.IH)

44C FORMAT (IX.' INITIAL BUDGET SURPLUSES :'./.9F13.6)

DO 130 I= 1.IH

IF (F(I).GT.0) GO TO 120

P(I)= 6(I) + F(I)/PI(I)

MFL AG= 1
229.
229.
251 .
252.
253.
256.
257 .
259.
259.
259 .5
                                             MFL AG=
                          130 CONTINUE

IF (MFLAG .GT. 0) GO TO 343

ZFLAG= 0

GO TO 460
260.
261 · 264 · 265 · 266 ·
                         ZFLAG= C
GO TO 460
450 NTEMP(1)= NTEMP(1)+ JTINV
CALL INVERT
CALL PECALC
ITSINV= 0
460 KEND= KFND+ 1
267.
268.
269.
270.
271.
                        273
274
275
278
279.
 230 .
281.
293.
 294.
295.
237.
288.
289.
290 ·
292 .
293.
                               A PRIMAL VARIABLE WENT TO ZERO. THAT VARIABLE MUST GO TO
NONBASIC. AND THE INCOMING VARIABLE IS DETERMINED BY FINDING THE
LARGEST ELEMENT IN THE APPROPRIATE ROW OF GI.
294.
295 ·
```

```
PD= -1
IR TWP= KINBAS(IS)
CALL FINDP(1.IS.JCCLP)
INNAM1= ICNAM(JCCLP.1)
INNAM2= ICNAM(JCCLP.2)
IONAM1= ICNAM(IS.1)
IONAM2= ICNAM(IS.1)
IONAM2= ICNAM(IS.2)
WRITC(6.485) INNAM1.INNAM2.IONAM1.IONAM2
485 FORMAT(IX.' VECIN: '.2A4.' VECCUT: '.2A4)
CALL PIVOT(JCCLP.IS)
CALL UNPACK(JCCLP)
CALL FIRAN(1)
CALL WRETA
298.
300.
301.
302.
303.
304.
306.
308.
                                               CALL WRETA
CALL BSCNG(JCOLP.IS)
CALL SUPERB(0.-1.JCOLP.1.JCOLP)
CALL BLCONS
 310.
311.
312.
313.
                                              JS= IS
IF (ITSINV .GE. INVFRQ) GO TO 450
SO TO 460
314 .
315.
316.
317.
313.
319.
                                        A DUAL VARIABLE WENT TO ZERO. THAT VARIABLE MUST ENTER THE BASIS, AND THE LEAVING VARIABLE MUST BE DETERMINED BY FINDING THE LARGEST PIVOT ELEMENT IN THE IDBAS(IS)-ROW OF G2.
32C.
 321.
                                 520 PD= 1

JCOLP= IS

CALL FINDP(-1.IS.P)

TROWP= KINEAS(P)
322 .
323.
324.
325.
326.
                                              TROWP= KINEAS(P)
INNAM1= ICNAM(IS.1)
INNAM2= ICNAM(IS.2)
IONAM1= ICNAM(P.1)
IONAM2= ICNAM(P.2)
WRITE (6.495) INNAM1.INNAM2.IONAM1.IONAM2
CALL PIVOT(JCOLP.P)
CALL UNPACK(JCOLP)
CALL WRETA
 327 .
 328.
329.
 331.
 332 .
 333.
                                              CALL FTRAN(1)
CALL WRETA
CALL BSCNG(JCOLP.P)
CALL SUPERB(0.1.P.-1.P)
CALL BLCONS
JS= IS
IF (ITSINV.3E.INVFRQ) GD TC 450
GO TO 460
334 .
 336.
337 .
338.
340.
 341.
                                    AND HER BILINEAR CONSTRAINT IS SATISFIED. WE INCREASE IN AND WE HAVE A POINT IN W(IH).
342.
343.
                                 220 WRITE (6.930) KEND
930 FORMAT(//,' AFTER'.I4,' ENCPNT CALLS. WE HAVE EQUILIBRIUM.')
WRITE (6.320) KFUN.KJAC
320 FORMAT(/,' TOTALS: SCALAR FUNCTION CALLS='.I8./.11X.'JACOBIAN EVA
1LUATIONS='.I8./)
CALL DEBUG(1)
DO 935 I= 1.IH
935 VS(1)=-8(!)+ XX(I)
WRITE (6.940) (VS(I).I=1,IH)
WRITE (6.945) (PI(I).I= 1, IH )
240 FORMAT (1X.' UTILITY LEVELS: '.6F15.7)
241 FORMAT (1X.' DUAL MULTPLIERS: '.9F12.6)
DO 950 I= 1.NROW
344 .
345 .
 346.
 347.
 348.
 349.
350.
351.
 352 .
 353.
 354 .
 355 .
 356 .
  157 .
```

```
1V= JH(1)
X(1)= XX(1V)
Y(1)= P1(1)
358.
359.
 36C.
 361.
                                                   CALL UNRAVE (0)
 363.
                                   1010 FORMAT (1814)
1030 FORMAT (1057.4 )
 364 .
 365.
366.
367.
                                                    END
                                                    SUBROUTINE BL CONS
 369.
                                         WE ARE GUING TO CALCULATE THE IH ROWS OF COEFFICIENTS FOR THE BILINEAR EQUATIONS AND THE BILINEAR INEQUALITY. THE BASIC MATHEMATICAL STRUCTURE IS THE FOLLOWING:
369 · 370 · 371 ·
 372.
                                                           BF1 + D1*PI(MU) - DIAG(PI(MU))*(F1*X(NU) + E1) = 0

BF2 + D2*PI(MU) - DIAG(X(NU))*(F2*PI(MU) + E2) = 0
 373.
374 · 375 · 376 ·
                               C
                                                    IMPLICIT REAL*8 (4-H.O-Z)
INTEGER PD.PD1.PD2.S.R.SS.RR.ZFLAG.RS.P
377 .
                                                   INTEGER#2 JH(350).DIGMA(952).KINEAS(1302).IDBAS(1302)
INTEGER#2 ISTYPE.LA.LE.IA.IE.PLN.LC(20).IC(800)
DOUBLE PRECISION E(8000)
REAL A(4000).C(800).CMIN.CCND.EPMAX.SUMINF
 378.
 379 ·
380 ·
 391 .
                                               COMMON DSUM.DPROD.DY.DE.CP.8(350).X(350).Y(350).YTEMP(350).

1A.F.CMIN.COND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).

2NTEMP(2C).KINP.ITIN.JTIN.ITINV.JTINV.MSTAT.IOBJ.IROWP.IVIN.IVOUT.

3I TCNT.INVERQ.ITRLIM.IFFEZ.JCOLP.NECW.NCOL.NELEM.NETA.NLELEM.NLETA.

4NGELEM.NINF.NUELEM.NUETA.NNEGDJ.NLINES.1STYPE(350).

5LA(1302).LF(2002).PUN(8).

61PUNC.NDEGI.NDUAL.NIPIW.IFEAS.IFCRSH

COMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTB

COMMON/BLCST/BF1(10).BF2(10).E1(10).E2(10).C.IC.LC

COMMON/PLCST/BF1(10).BF2(10).E1(10).E2(10).F2(9.10)

COMMON/PLCSTZ/D1(10.10).D2(9.10).F1(9.10).F2(9.10)

COMMON/INDXI/ NUH(10).MUH(10).NU(10).MU(10)

COMMON/INDXI/ NUH(10).MUH(10).NU(10).MU(10)

COMMON/INDXI/ JH.DIGMA.KINEAS.ICRAS

COMMON/DIM/ IH.N.M.KMU.KNU.MP1.NM.ITAIL

K1 = 0
  382.
                               C
  383.
384 . 385 .
 386 .
 387 .
 389.
369 .
 397.
 391.
 392.
 393.
 394.
  395.
396 · 397 · 398 ·
399 .
                                      COMMUNICATION

K1 = 0

K2 = 0

IF (KMU.FO.O) GO TO 100

DO 80 K = 1 .KMU

K1 = MU(K)

IF (KI.GT.IH) GO TO 80

IDD= KINBAS(K1)

IF (KNU.FO.O) GD TO 45

DO 40 I = 1.KNU

40 F1(K.T) = G1(IDD.I)

45 E1(K) = BA(IDD)

CALL UPAKC(K1)

DO 60 I = 1.KMU

DSUM = 0.0

IF (MU(I) .LE. M)

DO 50 J = 1.M

IDJ= IDBAS(J)

IF (IDJ .FQ. 0) GO T
400.
401.
403.
404 .
405.
406.
407 .
408.
409.
410.
411.
412.
413.
414 .
                                                                                                                       DSUM= Y(MU(I))
415.
417 .
                                                               IF (10J .EQ. 0) GO TO 50
```

```
DSUM= DSUM+ Y(J)*G2(IDJ.I)
419.
420.
421.
422.
                                50 CONTINUE

D1(K,I) = DSUM

60 CONTINUE
                               DETINUE

BF1(K)= 0.

DO 70 J= 1.0M

IF(IDBAS(J).E0.0) GO TO 70

RF1(K)= RF1(K)+ Y(J)*BB(IDBAS(J))

70 CONTINUE

80 CONTINUE
 423.
424.
 426 .
 427.
 420 .
                               NOW THE SECOND TYPE OF EQUATION IS CALCULATED BECAUSE PI(\mathcal{K}) IS BASIC.
429.
430 .
                             100 IF (KNU.EQ.0) RETURN
DD 145 K= 1.KNU
K2= NU(K)
IF(K2.GT.1H) GD TO 145
IDD= IDBAS(<2)
IF (KMU.LQ.0) GD TC 123
DD 120 I= 1.KMU
120 F2(K.I)= G2(IDD.I)
123 E2(K)= B9(IDD)
CALL UPAKC(K2)
IF (KMU.EQ. 0) GD TO 132
DD 130 I= 1.KMU
D2(K.I)= 0.0
TF (MU(I) .LE. M) D2(K.I)= Y(MU(I))
D0 125 J= 1.M
IF( IDBAS(J).EQ.0) GD TC 125
D2(K.I)= D2(K.I)+ Y(J)*G2(IDBAS(J).I)
125 CONTINUE
                        C
 432.
 433.
 434.
435 .
 436 .
 437.
 438 .
 439.
 440 .
 441 .
 442.
443 .
 444.
 445 .
446 .
 447.
448 .
                             125 CONTINUE

130 CONTINUE

132 SE2(K) = 0.

20 140 J= 1.M

IF (IDBAS(J).EQ.0) GD TO 140
 449.
450 .
 451 .
 452.
453 .
                             140 CONTINUE

140 CONTINUE

15 (K2.NF.IH) GO TO 145

145 CONTINUE
454.
455.
456.
457.
458 ·
459 ·
460 ·
                                        RETURN
                                       END
                                        SURFOUTINE FINDP(PD1.15.P)
461 .
                      00000
                            THIS SUBFOUTINE CHOOSES THE VARIABLE TO BECOME IMPLICITLY BASIC AS THE ONE WITH THE PIVOT ELEMENT LARGEST IN ABSOLUTE VALUE
 462.
463 .
 464.
465.
                                       IMPLICIT REAL*8 (A-H.O-Z)
INTEGER PD.PD1.PD2.5.R.SS.RR.ZFLAG.AS.P
INTEGER*2 JH(350),DIGMA(952).KINBAS(1302).IDBAS(1302)
COMMONZINCONSZG1(350.10).GZ(400.10).BA(350).BB(400)
COMMONZINDXIZ NUH(10).MUH(10).NU(10).MU(10)
COMMONZINDXZZ JH.DIGMA.KINEAS.IDBAS
COMMONZONZZZ JH.DIGMA.KINEAS.IDBAS
COMMONZDIMZ IH.N.M.KMU.KNU.MPI.NM.ITAIL
COMMONZTOLERZ TOLEZ.TOLBC.TOLCV.THETA,STPMX.STPRD
 467.
 458 .
469.
470 ·
471 ·
472 ·
 473.
                                       474.
475 .
476 .
```

```
478.
                                           IF (P.NE.O) GO TO 10
BIG= DABS(G1(1D0.1))
                                   P= NU(1)
GO TO 2C
10 COMP= DASS(GI(ID).II)
IF (COMP.LE.BIG) GO TO 20
48C.
481.
482 .
483.
484 .
                                           PIG= COMP
485.
                                           P= NU(I)
                                  20 CONTINUE
IF (BIG.LT.TOLFZ) P= 0
486.
487 .
488.
                                            RETURN
                                   30 IDD= IDBAS(IS)
489 .
                                           DD 40 J= 1.KMU

IF (P.NE.O) 30 TO 35

PIS= DARS(G2(IDD.J))

P= MU(J)
490.
191.
492 .
493.
                                           TOMP= DABS(G2(IDD.J))
IF (COMP.LE.BIG) GO TO 40
BIG= COMP
P= MU(J)
494 .
495.
496.
497 .
498.
                                   40 CONTINUE
                                           IF (BIG.LT.TOLFZ) P= 0
RETURN
499.
500.
501.
                                           END
502 .
                                           SUBROUTING PIVOT (SS.RR)
503.
                          00000
504 •
                                   THIS POUTINE PERFORMS A PIVOT ON THE PRIMAL SUPERBASIC COLUMNS IF PO.EQ.1. DUAL IF PD.EQ.-1. THE PIVOT BEINGS COLUMN SS INTO THE EASIS AND COLUMN RE OUT OF THE
506.
507.
                                   BASIS.
508.
                                           IMPLICIT REAL*8 (4-H.D-Z)
INTEGER PD,PD1.PD2.S.R.SS.RR.ZFLAG.RS.P
INTEGER*2 JH(350).DIGMA(952).KINRAS(1302).IDBAS(1302)
INTEGER*2 ISTYPE.LA.LE.TA.TE.PUN.LC(20).TC(800)
DOUBLE PRECISION E(8000)
507.
510.
511.
512.
513.
514.
515.
516.
                                                           A (4000) .C(300) .CMIN . COND . ERMAX .SUMINF
                                       COMMON DSUM.DPROD,DY.DE.DP.3(350),X(351),Y(350),YTEMP(350),

14.E.CMIN.COND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20),

2NTEMP(2C).KINP.ITIM.JTIM.ITINV.JTINV,MSTAT.IOBJ.IROWP.IVIN.IVOUT.

3I TCNT.INVFRQ.ITRLIM.IFFEZ.JCOLP.NRCW.NCOL.NELEM.NETA.NLELEM.NLETA.

4NGELEM.NINF.NUELEM.NUETA.NNEGDJ.NLINES.ISTYPE(350).

5LA(1302).LE(2002).FUN(8).

6IPUNC.NDEGI.NDUAL.NIPIW.IFBAS.IFCRSH
COMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTB
COMMON IA(4000).IE(8000)
COMMON/LPI/PI(1302).XX(1302)
COMMON/LNCONS/GI(350.10).G2(400.10).BA(350).BB(400)
COMMON/INDX1/NUH(10).MUH(10).NU(1C).MU(10)
COMMON/INDX2/JH.DIGMA.KINEAS.ICBAS
COMMON/DIM/IH.N.M.KMU.KNU.MPI.NM.ITAIL

FPS= 16.**(-13)
RH= KINPAS(RR)
IF (5S.GT.IH) GO TO 5
518.
519.
520.
521.
522 .
523.
524 · 525 · 526 ·
528 .
529.
530 .
531 · 532 ·
                                     ### KINGAS(RR)

IF (SS .GT. IH) GO TO 5

IF (NUH(SS).NE.0) GO TO 20

5 CALL UNPACK(SS)

CALL FTRAN(1)

YTEMP(RE) = -1./Y(RB)

DO 10 I = 1.M
533 .
534 .
535.
536.
538 .
```

```
IF (1.NE.RB) YTEMP(1) = Y(1)*YTEMP(RB)
10 CONTINUE
GO TO 30
20 JS= NUH(95)
539.
540.
541.
                                        20 JS= NUH(SS)
    YTEMP(RB)= 1./G1(RB.JS)
    DO 25 I = 1.M
    IF (I.NF.RB) YTEMP(I)= -G1(I.JS)*YTEMP(RB)
25 CONTINUE
30 IF (KNJ.E0.0) GO TO 55
    OO 50 J= 1.KNU
    IF (NU(J).EQ.SS) GC TO 50
    V= G1(RB.J)
    G1(RB.J)= 0.
    DO 40 I= 1.M
40 G1(I.J)= G1(I.J)+ V*YTEMP(I)
    G1(RB.J)= -G1(RB.J)
50 CONTINUE
50 V= BA(RB)
9A(RB)= 0.
542.
543.
 544 .
 545 .
 546.
 547 .
548.
550 .
 551 .
552.
553.
554.
555 ·
                                          55 V= BA(RB) = C.

BA(RB) = C.

BO 60 I = 1.M

60 BA(I) = RA(I) + V*YTEMP(I)

BA(RB) = -BA(RB)
 557.
558.
559.
560.
561.
                                00000
562.
563.
564.
                                          WE ARE NOW PIVOTING ON THE DUAL SYSTEM. IF THE IS NOT AN IMPLICIT BASIC-TYPE PIVOT, WE CALL SUPERB TO CALCULATE THE PIVOT COLUMN.
 566.
 567.
                                                    RB= IDBAS(SS)
IF (RR .GT. IH) GO TO 70
IF (MUH(RR).NE.0) GO TO 80
568 .
569.
570.
571.
572.
                                         70 IFL= 1
CALL SUPERB(1.-1.RR.0.0)
JS= KMU
GU TO 52
                                     JS= KMU
GD TD 82

8C JS= MUH(RR)
32 YTEMP(RB)= 1.7G2(RB.JS)
DO 95 I= 1.N
IF (I.NE.RB) YTEMP(I)= -G2(I.JS)*YTEMP(RB)

85 CONTINUE
IF (KMU.FQ.O) GO TC 115
DD 110 J= 1.KMU
IF (MU(J).EQ.RR) GC TO 110
V= G2(RB.J)= 0.
DD 100 I= 1.N

100 G2(I.J)= G2(I.J)+ V*YTEMP(I)
G2(RB.J)= -G2(RB.J)

110 CONTINUE
115 V= BB(RB)
BR(PB)= 0.0
DD 120 I= 1.N

120 BB(RB)= -BB(RB)
IF (IFL.EQ.O) RETURN
CALL SUPERB (2.0.0.-1.RR)
RETURN
ENO
573.
574.
575.
576.
577.
578 · 579 ·
580 .
 581 .
 582.
593.
 584.
585 ·
586 ·
 599.
 589.
59C •
592.
593.
 534 .
595 .
596.
597.
598.
                                CC
```

2

```
SURROUTINE UPAKC(II)
IMPLICIT REAL*8 (A-H.O-Z)
INTEGER*2 JH(350).DIGMA(952).KINBAS(1302).IDBAS(1302)
INTEGER*2 ISTYPE.LA.LE.IA.IE.PUN.LC(20).IC(800)
DOUBLE PRECISION E(8000)
REAL A(4000).C(300).CMIN.CGND.ERMAX.SUMINE
599.
600.
601.
503.
504 .
605 .
                          C
606.
                                           COMMON DSUM.DPROD.DY.DE.DF.8(350),X(350),Y(350),YTEMP(350).
                                      COMMON DSUM.DPROD.BY.DE.DP.8(350).X(350).Y(350).YTEMP(350).

1A.E.CMIN.COND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).

2NTEMP(20).KINP.ITIM.JTIW.ITINV.JTINV.MSTAT.IOBJ.IROWP.IVIN.IVDUT.

3I TONT.INVFRQ.ITRLIM.IFFEZ.JCOLP.NROW.NCOL.NELEM.NETA.NLELEM.NLETA.

4NGFLEM.NINF.NUELEM.NUETA.NNEGDJ.NLINES.ISTYPE(350).

5LA(1302).LF(2002).PUN(8).

6IPUNC.NDEGI.NDUAL.NIPIW.IFBAS.IFCRSH

COMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTB

COMMON IA(4000).IE(3000)

COMMON/BLCST/BF1(10).BF2(10).E1(10).E2(10).C.IC.LC
507 .
SCA.
509.
611.
 513.
614 .
615.
616.
                          C
 518.
                                          00 100 I= 1.M
                              100 CONTINUE
619.
 521 .
                         C
                                          LL= (C(11)
KK= LC(11+1) -
DD 200 I= LL.KK
IR= (C(1)
622 ·
623 ·
524 .
526.
                                           Y(19)=
                                                             C(I)
 527 .
                              200 CONTINUE
623.
                         C
629.
                                          HETUEN
631.
                         CC
                                 THE FOLLOWING ROUTINE MAKES THE CHANGES IN THE INDEX SETS NECESSARY EVERY TIME A BASIS CHANGE IS MADE.
532 .
533 .
634 .
                                          SURROUTINE BSCNG(S.R)
INTEGER#2 JH(350),DIGMA(952).KINBAS(1302),IDBAS(1302)
COMMONZINOXZZ JH.DIGMA.KINBAS.IDBAS
635 .
637 .
638.
                                           INTEGER
                                                                 S.SSAV.R.FSAV
                                          RSAV= KINBAS(R)
JH(RSAV)= S
KINBAS(F)= 0
KINBAS(S)= RSAV
639 .
540 .
541 .
642 .
543.
                                           SSAV= IDBAS(S)
                                          DIGMA(SSAV) =
644 .
645.
                                          108AS(S)= ?
108AS(R)= 3SAV
647 .
                                           RETURN
548.
                                           FND
                                THIS SURROUTINE CAN DO THREE THINGS DETERMINED BY THE PARAMETER 'KEY'. IT CAN ADD A COLUMN. REMOVE A COLUMN. OR BOTH ADD AND GEMOVE COLUMNS FROM THE PRIMAL OR DUAL SUPERBASIC COLUMNS. DEPENDING ON WHETHER KEY IS 1. 2. OR 0. RESPECTIVELY. POI IS 1 OR -1 DEPENDING ON WHETHER A PRIMAL OR DUAL COLUMN IS REING ADDED. POZ IS A SINILAR FLAG FOR THE COLUMN BEING REMOVED. IS AND JS INDICATE THE PARTICULAR COLUMN TO BE ADDED OR REMOVED FROM THE GROUP OF COLUMNS SPECIFIED BY POI AND POZ.
649.
650 .
552 .
                         00000
653.
654 .
655.
657 .
658.
```

1

```
SUBROUTINE SUPER9(KEY,PD1.IS.PD2.JS)

IMPLICIT REAL*8 (A-H.D-Z)

INTEGER PD.PD1.PD2.S.R.SS.RR.ZFLAG.RS.P

INTEGER*2 JH(350).DIGMA(952).KINBAS(1302).IDBAS(1302).

INTEGER*2 ISTYPE.LA.LE.IA.IE.PUA.LC(20).IC(800)

DOUBLE PRECISION E(8000)

PEAL A(4000).C(900).CMIN.CGND.ERMAX.SUMINF
659.
550.
651.
662.
663.
664 .
665.
                             666.
                   C
667.
668 .
667.
670.
671 .
672.
674 .
675.
676.
578.
579 .
600.
681 .
682 .
683.
684 .
685.
696 .
                                KMU= KMU- 1
IF (JS .GT. 1H) GC TO 4
MH= MUH(JS)
587 .
688.
689 .
59C .
                                 MUH(JS)= 0
                               IF (MH.EQ.KM) GD TO 6
00 5 != MH.KMU
1M= MU(I+1)
MU(I)= IM
MUH(IM)= MUH(IM)- 1
691 .
692 ·
594 .
695 .
                                00 5 J= 1.N
G2(J.1) = G2(J.1+1)
696.
697 .
                         G2(J.I) = G2(J.I+1)
5 CDNTINUE
6 MU(KM) = 0
4 DD 7 J= 1.N
7 G2(J.KM) = 0.0
IF (KEY.EQ.2) RETURN
GD TO 50
20 KN = KNU
KNU = KNU - 1
IF (JS.GT. IH) GD T
698.
699 .
700.
701 .
702.
703.
704 .
705.
                                IF (JS .GT. IH) GO TO 28

NH= NUH(JS)

NUH(JS)= 0

IF (NH.EQ.KN) GO TU 26

DO 25 I= NH.KNU
706.
707.
708.
709.
710 .
                                 JN= NU( 1+1 )
                                DO 25 J= 1.M
G1(J,I)= G1(J,I+1)
712.
713.
715.
716.
                          25 CONTINUE
                          26 NU (KN) = 0
28 30 30 1 = 1 · M
30 G1(1 · KN) = 0 · 0
717 .
718.
```

```
720 • 721 • 722 • 723 • 724 •
                                    IF (KEY.EQ.2) RETURN
                                       NOW WE ADD THE SUPERBASIC VARIABLE SPECIFIED BY PDI . IS .
                            50 IF (PD1.LT.0) GO TC 70

KN= KNU

KNU= KNU+ 1

IF (15.GT.IH) GO TO 55

NU(KNU)= IS

NUH(15) = KNU

55 CALL UNPACK(IS)

CALL FTPAN(1)

DO 60 I= 1.M

60 G1(T.KNU)= -Y(I)
 725 .
 726.
 727.
728 · 729 · 730 · 731 ·
732.
733.
734.
735.
                             40 G1(I.KNU)= -Y(I)
 736 .
                                         NOW WE ARE BRINGING & DUAL SUPERBASIC COLUMN IN.
 737.
739.
739.
740.
                             70 KMU= KMU+ 1
1F (19.6T.1H) GO TO 73
MU(KMU) = IS
                             MUH(15) = KMU
73 IF (15.LE.M) GO TO 110
 741 .
 742.
 743.
                             THE ENTERING VARIABLE IS A ZETA. THE
                                                                                                                         THE ENTER ING COLUMN
744 . 745 .
746.
                             IDD= KINBAS(IS)
DD 75 I= 1.M
75 Y(I)= C.
Y(IDD)= 1.
748 .
749.
                          Y(100)= 1.

CALL PTRAN
CALL SHIFTR(3.2)
DD 90 I= 1.M
IF (108AS(I).EQ.0) GD TO 80
G2(ID8AS(I).KMU)= X(I)
80 CONTINUE
DD 100 J= MP1.NM
IF (108AS(J).EQ.J) GD TO 100
CALL UNPACK(J)
DDT= 0.0
DD 90 I= 1.M
IF (108AS(I).EQ.O) GD TO 90
DT= DDT+ Y(I)*X(I)
90 CONTINUE
G2(INSAS(J).KMU)= DCT
CONTINUE
RETURN
751 .
 752 .
753.
754 · 755 ·
 756 .
 757.
758.
 760.
751 .
762.
763.
764 .
 765.
766 · 767 ·
                                    RETURN
 768.
                           THE ENTERING VARIABLE IS A PI. THE ENTERING COLUMN IS PARTLY CI*INV(B) AND PARTLY C2+ C1*INV(B)*D.
-C1*INV(B) IS PART OF THE EASIS INVARSE CORRESPONDING TO THE X-BASIC COLUMNS AND THE T-BASIC ROWS.
769.
770.
 771 .
772.
773.
774.
775.
                           110 100= KINBAS(IS)
                           110 100= KINHAS(IS)
20 130 I= 1.M
130 Y(I)= C.
Y(IDD)= 1.
CALL STRAN
CALL SHIFTR(3.2)
776.
 778.
779.
```

```
780 · 781 · 782 · 783 · 784 ·
                                        00 140 I= 1.M
IF (108AS(I).E0.0) GO TO 140
S2(IDBAS(I).KMU) = X(I)
140 CONTINUE
                                       14C CONTINUE
DO 160 J= MP1.NM
IF (IDBAS(J).EQ.0) GO TO 160
CALL UNPACK(J)
DOT= 0.0
DO 150 != 1.M
IF (IDBAS(I).EQ.0) GO TO 150
DOT= DOT+ Y(I)*X(I)

150 CONTINUE
$\frac{2}{2}(IDBAS(I).KMU) = Y(ISA+ COI)$
 785 .
 786.
788 .
 789.
 740.
 701 .
                                        SZ(IDBAS(J).KMU) = Y(IS)+ COT
 792.
793.
 794 .
                                                       RETURN
 795 .
                                                       FND
 796.
797.
                                                       SUBFOUTING RECALC
                                       THIS SUBROUTINE RECALCULATES THE SUPERBASIC COLUMNS USING THE CURRENT BASIS IN GR FORM. IT CAN ALSO ADD A VARIABLE AND COLUMN TO THE SUPERBASICS.
 798 .
 799.
                                 C
 900 .
801 .
                                                      IMPLICIT REAL*8 (A-H.O-Z)
INTEGER PD.PD1.PD2.S.R.SS.RR.ZFLAG.RS.P
INTEGER*2 JH(350).CIGMA(952).KINEAS(1302).IDBAS(1302)
INTEGER*2 ISTYPF.LA.LE.IA.IE.PUN.LC(20).IC(800)
DOUBLE PRECISION E(800).CVIN.CCND.ERMAX.SUMINF
802.
 304 .
 3C5.
 807.
                                          COMMON DSUM.DPROD.DY.DE.CF.B(350).X(350).Y(350).YTEMP(350).

1A.E.CMIN.CUND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).

2NTEMP(20).KINP.ITIM.JTIM.ITINV.JTINV.MSTAT.10BJ.IROWP.IVIN.IVOUT.

3ITCNT.INVERG.ITELIM.IFEEZ.JCOLP.NCCW.NCUL.NELEM.NETA.NLELEM.NLETA.

4NGELEM.NINF.NUELEM.NUETA.NNEGDJ.NLINES.ISTYPE(350).

6IPUNC.NDEGI.NJUAL.NIPIW.IFBAS.IFCRSH

CDMMON ITCH.ITCHA.IFPIWT.IFNEG.KOUTB

COMMON/LPI/PI(1302).XX(1302)

COMMON/LPI/PI(1302).XX(1302)

COMMON/INDXI/ NUH(10).MUH(10).NUL(10)

CDMMON/INDXI/ NUH(10).MUH(10).NUL(10)

CDMMON/SCAL/ BT.NR.JJ.MFLAG.IHPI.P.PD.MPD.KFUN.KJAC

COMMON/SCAL/ BT.NR.JJ.MFLAG.IHPI.P.PD.MPD.KFUN.KJAC

COMMON/OIM/ IH.N.M.KMU.KNU.MPI.NM.ITAIL

EPS= 16.**(-13)

IF (KNU.EQ.O) GO TO 26

DD 28 J= 1 KNU

IDD= NU(J)

CALL UNPACK(IDD)

CALL FTRAN(I)

DO 25 I= 1.M

25 G1(I.J) = - Y(I)

28 CONTINUE

6 CALL SHIFTR(1.3)
808 .
                                 C
810 .
 311.
313.
 914.
 815.
 a17.
918.
821 .
 922.
 923.
824.
 825 .
 828 .
930.
831 .
 932.
                                            28 CONTINUE

26 CALL SHIFTR(1.3)

CALL FTRAN(1)

DO 22 I = 1.M

22 3A(1) = Y(1)
 933.
434 .
 935.
936 .
 337 .
                                                       IF (KMU.ED.O) GO TO 140
DO 130 J= 1.KMU
TOD= MU(J)
 939 .
                                            30 IF
939 .
 940.
```

```
941.
942.
843.
944.
                                            IF (IDD .La .M) GO TC 70
                                            ZETA- VARIABLE IS ENTERING
                                 IB= KINPAS(IDD)

DD 35 I = 1.M

35 Y(I) = 0.
    Y(IF) = 1.
    CALL BTRAN
    CALL SHIFTR(3.2)
    DD 40 I = 1.M
    IF (IDNAS(I).EQ.0) GD TO 40
    G2(IDRAS(I).J) = X(I)

40 CONTINUE
    DD 60 K= MP1.NM
    IF (IDRAS(K).EQ.0) GD TO 60
    CALL UNPACK(K)
    DDT = 0.0
    DO 50 I = 1.M
    IF (IDRAS(I).EQ.0) GD T) EC
    DDT = DOT + Y(I) * X(I)

50 CONTINUE
    G2(IDRAS(K).J) = DDT
60 CONTINUE
    GO TO 130
 944 .
                                            IB= KINPAS(100)
 34 € . .
 347 .
 940 .
 94 7.
 950 .
951.
952.
953.
 954.
 355 .
 956.
55A .
 350 .
 961.
962.
 964.
 965.
                                           GO TO 130
 966 .
                                           PI - VARIABLE IS ENTERING
868 .
 869.
                                   70 18= KINBAS(IDD)

00 90 I= 1.M

90 Y(1)= 0.
 970 .
 971.
                               90 Y(1)= 0.

Y(1P)= 1.

CALL STEAN

CALL STIFTR(3.2)

DD 100 I= 1.M

IF (1DPAS(I).EQ.0) GD TD 100

G2(IDRAS(I).J)= X(I)

100 CONTINUE

DD 120 K= MP1.NM

IF (1DPAS(K).EQ.0) GD TC 120

CALL UNPACK(K)

DDT= C.0

DD 110 I= 1.M

IF (IDBAS(I).EQ.0) GD TC 110

DDT= DDT+ Y(I) *X(I)

110 CONTINUE

G2(IDPAS(K).J)= Y(IDD)+ DDT
872.
873.
874.
 976 .
 977.
970.
 98C .
 891.
883.
 995
                              39A.
 897.
 999.
 949.
 390 .
 891 .
892.
 394.
 995 .
 995.
 999 .
 399.
  200 .
```

```
DD 165 T= 1.M

IF (108AS(I).EQ.0) GO TC 165

DDT= DDT+ X(I)*Y(I)

165 CONTINUE
901.
904 .
                                 170 CONTINUE
905.
906.
                                              RETURN
90A .
                                              END
                                              SUBPOUTINE ENDPHT (JS.PD1.IS.ZFLAG.NET)
IMPLICIT REAL *8 (4-H.O-Z)
REAL *8 MIN.MIN2
REAL *4 C(800)
909.
910 ·
911.
                                             REAL*4 C(800)
INTEGER PD.PDI.PD2.S.R.SS.RR.ZFLAG.RS.P
INTEGER*2 JH(350).DIGMA(952).KINBAS(1302).IDBAS(1302)
INTEGER*2 JSTYPE.LA.LE.IA.IE.PUN.LC(20).IC(800)
COMMON/NEWT/ H(10.11).X(10).Z(10).ACC(3.10).BLAM(10)
COMMON/PLCST/EF1(10).BF2(10).E1(10).E2(10).C.IC.LC
COMMON/BLCST/EF1(10.10).D2(9.10).F1(9.10).F2(9.10)
COMMON/BLCSTZ/D1(10.10).D2(9.10).F1(9.10).F2(9.10)
COMMON/INCONS/G1(350.10).G2(400.10).BA(350).BB(400)
COMMON/INDXI/ NUH(10).NUH(10).NU(10).MU(10)
COMMON/INDXI/ NUH(10).NUH(10).NU(10).MU(10)
COMMON/SCAL/ BT.NP.JJ.MFLAG.IHP1.P.PD.MPD.KFUN.KJAC
COMMON/DIM/ IH.N.M.KMU.KNU.MP1.NM.ITAIL
COMMON/INT/ IPS(30).KDET.KDUNT.ISING.KEND
COMMON/TOLE Z/ TOLE Z.TOLE L.T JCCV.THETA.STPMX.STPRD
DIMENSION U(3).V1(10).V2(10).F(10).DOT(4).RHS(10).UL(10.10)
DIMENSION G(10)
¥13.
915.
916.
918.
919.
921.
 923.
924 .
725.
126.
 120.
929.
                           OCCOCCOO
                                       THIS SUBROUTINE WILL FIND THE OPPOSITE ENDPOINT OF G**(-1)(0) WITH RESPECT TO THE CELL DEFINED BY THE CURRENT BASIS. WHERE
130 ·
131.
                                                                G(X.THETA) = THETA*F(X) - (1-THETA)*X
F(X) = THE BUDGET SURPLUSES DETERMINED BY THE
FIN AND BLCJNS SUBROUTINES.
THETA = THE HOMOTORY FARAMETER.
 933.
934 .
935.
936.
 93A.
                                              NET= 0
                                              INTLE OFRACEO.5
938.5
939.
                                              DELTA = .01
TWDEL= 2.0 MOELTA
94C .
141 .
 742 .
                                       MFLAG=0 WHEN WE ARE SOLVING FOR THE QUADRATIC APPROXIMATION
MFLAG=1 WHEN WE ARE SOLVING ONE OF THE HYPERPLANE SUBPROBLEMS.
MFLAG=2 WHEN DNE OF THE LINEAR CONSTRAINTS IS BINDING.
MFLAG=3 WHEN THE EQUILIBRIUM POINT APPEARS TO BE IN THE CURRENT
CELL. NEWTON'S METHOD WILL BE IMPLEMENTED AS A TAIL ROUTINE ON F.
 343.
944 .
945.
946 .
948.
 749.
                                              MFL AG=0
 950 .
                                              KK= 0
951.
                                               IHP1 = IH+
 753.
                                              = ( [HP1) = 0.0
                                              KMU1= KMU+
954 .
255.
                                ICX= THPI

INITIALIZE THE INDEPENDENT VARIABLES IN X.

IF (KMU.60.0) GO TO 615

00 610 I= 1.KMU

610 X(I)= PI(MU(I))
956 .
957.
358,
159 .
```

```
615 IF (KNU.EQ.0) GO TO 625

DO 620 I= 1.KNU

KM= KMU+ I

620 X(KM)= XX(NU(I))

X(IHPI)= THETA

ITER= 0
 960 .
961 .
962 .
963 .
964 .
                        | TFR = C
| SGCT = C
| STER = ITFR + I
| CALL GTN(D.G.X.F)
| CALL DERIVG(D.X.G.F)
| TISGCT = ISGCT + I
| IF (ISGCT.GT.IH) GD TD 300
| DD 10 I = I.IH
| PHS(I) = -H(I.ICX)
  346.
 967.
96F
 959.
970.
971.
972.
 973.
                           376.
 97A .
  970.
 980 .
  ...
                                    SOLVE THE LINEAR SYSTEM TO FIND THE TANGENT TO THE CURVE DEFINED BY G**-1(0).
  782.
  P.F.AC
  234 .
                           285.
 . 440
  987.
  . SRC
  . 020
  300.
                         GO TO 7

38 CALL SOLVE(IH.UL.RHS.V2)

40 DO 39 I = 1.IH

IM= IHP1- I

IF (IM.LT.ICX) GO TO 41

IP1= IM+ 1

V2(IP1)= V2(IM)

79 CONTINUE

41 V2(ICX)= 1.0

IF (ITFR.GT.1) GO TO 45

SUM= 0.0

IF (PD.FQ.-1) GO TO 110

JR= KINBAS(JS)

IF (J8.NE.0) GU TO 104

IF (V2(ID).GT.0) GO TO 135

GO TO 140

104 00 105 I= 1.KNU

IK= KMU+ I
  291 .
  305.
  993.
  994 .
  225.
  196.
  907.
  . Rpp
  999.
1 200 .
1001
1004.
1006.
                                             1K = KMU+ 1
SUM= SUM+ V2(1K)+G1(JE.1)
100A .
                         1000.
1011.
1013.
  014.
1715.
1015.
1012 .
                          115 CONTINUE
1010.
```

```
1021.
                                      INIT IS THE CURVE INDEX; IT TELLS US WHICH DIRECTION THE TANGENT MUST POINT TO KEEP GOING IN THE SAME DIRECTION.
                              IF (SUM.LT. 0.0) GO TO 140

135 INIT= KDET
GO TO 45

140 INIT= -KDET
45 CALL NORM(V2.S1.IHP1)
S1= (KDET*INIT)/S1
DO 47 I= 1.IHP1
ACC(2.I)= V2(I)*S1
ACC(3.I)= X(I)

47 CONTINUE
WRITE (6.850)
DO 48 J= 1.IHP1
WPITE (5.855) F(J). (ACC(I.J).I=2.3)

49 CONTINUE
1024
1025
1026
1027
1027
1029
1030
1033.
1034 · 1035 · 1036 ·
 1037.
                                  49 CONTINUE
                          000
 1039.
1039.
                                          FIND THE FIRST BOUNDARY THAT THE TANGENT. Q(ALPHA) HITS. WE ARE TRYING TO FIND THE SMALLEST ALPHA SUCH THAT G(I,.)*O(ALPHA) + GB(I) = 0
                          COO
1041.
 1042.
 1043.
                                          IFLAG= 0
IF (KNU.EQ.O) GU TO 249
DO 250 [= 1.M
LV= JH(])
 1044.
1045.
 1947.
                                          1048.
1049.
1050.
1051.
1052.
1053.
                                                    U(LL) = U(LL) + G1(I.K) *ACC(L)

CONTINUE

(GNTINUE

U(3) = U(3) + 3A(I)

IF (DARS(U(2)).LT.TOLFZ) GU TO 250

ALPHA = -U(3)/U(2)

IF (ALPHA .LT. -T)LFZ) GO TO 250

IF (ALPHA .GT. TOLRC) GO TO 244

TOO = 0.0

DO 242 K = 1.KNU

KM = K + KMU

TOD = TOD + G1(I.K) *ACC(2.KM)

IF (TDD .GT. -TOLFZ) GO TO 250

IF (IFLAG.EQ.1) GO TO 243

IFLAG = 1

JJ = T
1054.
                                230
                                241
 1957 .
 1058.
1059 .
1260.
1262.
 1963.
  264 .
                               242
1765.
1067.
                                344
 1968.
                                                      JJ= 1
 1070.
                                                     MIN= ALPHA
                                                     MPD= 1
GO TO 250
I= (ALPHA.GF.MIN) GO TO 250
1277.
                                243
                               243 | F (ALPHA.GF.MIN) GO TO 250

JJ= I

MIN= ALPHA

MPD= 1

250 CONTINUE

240 | F (KMU.EG.0) GO TO 640

70 250 I= 1.N

17 (DIGMA(I) .EJ. M) GC TO 259
 1073.
   075 .
 1074 .
 1078.
```

1270.

```
DO 256 LL= 2.3

U(LL)= 0.0

DO 254 K= 1.KMU

U(LL)= U(LL)+ G2(1.K)*ACC(LL.K)

CONTINUE
1080.
1081 .
1292.
1984 .
1095.
                         25€
                                         CONTINUE
                                         CONTINUE
U(3) = U(3) + BB(I)
IF (DABS(U(2)).LT.TCLF2) GO TO 259
ALPHA = -U(3)/L(2)
IF (ALPHA .LT. -TOLF2) GO TO 259
IF (ALPHA.GT. TOLHD) GC TO 258
TOD= 0.0
DO 252 K= 1.KMU
TOD= 100+ ACC(2.K)*G2(I.K)
CONTINUE
  CP6 .
                         25.1
1097.
1088.
1090.
1091.
1292.
1075.
1294 .
                         252
                                          IF (TUD .GT. -TCLFZ) GC TO 259
IF (IFLAG.EQ.1) GU TO 253
1000.
1096 .
1097.
                                          IFLAG = 1
                                         JJ= I

MIN= ALPHA

MPD= -1

GO TJ 259

IF (ALPHA.GE.NIN) GC TO 259
1098 .
1299.
1100 .
1101.
                        253
                                         JJ= 1
MIN= ALPHA
  104 .
1105.
1106.
1107.
1109.
1109.
                                          MPD= -1
                        259 CONTINUE
540 MFLAGE 2
                                      (KEND.GT.1 .09. INFL.EQ.1) GC TC 251
MIN= MIN* 0.25
                                 IF
1110.
                                          MELAGE
                                          INFL= 1
                         535 FURMAT (1X. WE MOVE A DISTANCE ".F15.6." ALONG THE APPROX. "./.
1' THE CURVE HIT A ".I3." TYPE CONSTRAINT, NUMBER ".I3)
1112.
1114.
                                 SET THE INITIAL X FOUAL TO Q(ALPHA) AS AN APPROX-IMATION TO THE ENDPOINT SOLUTION.
                    C
1116.
                         251 WRITE (6.635) MIN. MPD. JJ

IF (MIN.LT. STPMX) GO TO 255

MINE MIN* STPRC

MFLAG = 1
 1118.
1118
1119
1120
1121
1122
1123
1124
                                       MFLAG = 1
WRITE (6.635) MIN. MPD. JJ
                        1125
1126
1127
1128
1120
1130
1131
                        290 CONTINUE
WRITE (6.292) MFLAG
292 FORMAT(/.1x. MFLAG= '.14)
IF (MP) .5Q. -1) GO TO 293
IF (JH(JJ) .6Q. NM) GO TO 295
293 IF (ITAIL .6Q. 1) GO TO 295
264 TE 0
1133.
                                 TTRY= ITRY+ 1
ITRY= ITRY+ 1
IF (ITRY .LE. 6) GC TO 265
ITAIL= 1
GC_TO 260
1134.
1136
1137
1139
                        265 CALL GTN(MFLAG.G.X.F)
CALL NORM(G.S1.14P1)
1139 .
```

```
WRITE (6.935) IT. (X(I).I=1.IHP1)
WRITE(A.936) SI.(G(J).J=1.IHP1)
CALL DERIVG(MFLAG.X.G.F)
IF (SI.LT.TJLCV) GC TJ 280
IT= IT+ 1
NET= NET+ 1
IF (IT.LE.10) GD TC 265
MFLAG= 1
MIN= MIN/2.
IF (MIN .GT. TCLPD) GC TD 255
STOP
1140 ·
1141 ·
1142 ·
1143 ·
1144.
1145 .
1146.
1147.
1148.
                                    TITE MIN'S.

IF (MIN'ST. TOLRD) GC TO 255

STOP

366 IF (NET **LT. 60) GC TO 268

269 WRITE (6.267)

267 FORMAT (* ENDPOINT HAS FAILED --- TOO MANY ITERATIONS.*)
 1150.
1151.
1152.
1153.
                                                 STOP
                                    268 CALL DECOMP(IHP1.H.UL)

IF (ISING.E0.1) GC TO 300

CALL SOLVE(IHP1.UL.G.Z)

50 270 I= 1.IHP1

X(I)= X(I)- Z(I)
 1155.
 1156.
1157.
 1150.
                                    GO TO 265
280 IF (X(IHP1) .GT. 1.0001) GC TO 295
IF (MFLAG.EQ.2) GO TO 800
GO TO 625
1160 .
1161.
1162.
1163.
1164.
                              OF THE CUR.

295 MFLAG= 3

DO 100 K= 1.21

KM1= K- 1

CALL FTN(F.X)

CALL NOTM(F.S1.IH)

WRITE (6.93A) KM1. (X(1).I=1.IH)

WRITE (6.939) S1. (F(1).I=1.IH)

IF (S1.LT.TOLEZ) G) TO BOO

CALL DERIV

NET= NET+ 1

IF (IH.GT.1) GO TO 55

IF (DABS(H(1.1)) .LT. TOLEZ) GO TO 58

Z(1)= F(1)ZH(1.1)

GO T.65

DECOMP(IH.H.UL)
                                          NEWTON'S METHOD FOR THE PROBLEM OF FINDING THE INTERSECTION OF THE CURVE WITH THE CONSTRAINT DEFINED BY MPD AND JJ.
1165.
1166.
1169.
1169.
1170.
 1171.
1172.
1173.
1174.
1175 .
1176.
1177.
1176.
1179.
 1181.
1182.
1184.
1185.
                                                               CALL SOLVE(IH,UL,F,Z)
ALPH= 1
IF (ITAIL,F).C) GD TD 80
CALL DSFNT(ALPH.51)
IF (ALPH.GT. TOLCV) GC TD 70
WRITE (6,943)
GC TD 3CC
WRITE(6.955) ALPH
DC 95 I= 1.IH
X(I) = X(I) - ALFH*Z(I)
                                       65
1182.
1190 .
1191.
1192.
                                       70
1194.
                                       90
                                    95 CONTINUE
100 CONTINUE
30 TO 300
300 CALL CONCHK(GMIN.KGMIN.MP2)
1195.
 1198.
1192.
```

```
1200 .
1203.
1204.
1205.
1206.
                                                                         TO 148
1207.
                                               KM= NU(IK)
1208.
                           1209.
1210.
1211.
1212.
                                    ZFLAG= 2
1213.
                                    RETURN
                           RETURN

153 IF (MFLAG.EQ.1) 3D TO 150

155 IF (MPD.FQ. -1) GD TO 151

IS= JH(JJ)

PD1= 1

RETURN
1214.
1215.
1216.
1217.
1219.
                           151 IS= DIGMA(JJ)
PD1= -1
RETURN
1210.
1221.
1222.
1223.
1224.
1225.
1226.
                           150 MELAGE 0
                               IF THE CURRENT VALUE OF X VIOLATES THE CONSTRAINT DEFINED BY KGMIN AND MP2. FIND A GOOD STARTING POINT FOR NEWTON'S METHOD BY FINDING THE POINT ON THE LINE SEGMENT ACC(1..) + ALPHA*(X - ACC(1..)). O<= ALPHA<= 1 THAT SATISFIES THE VIOLATED CONSTRAINT EXACTLY.
1227.
1227 ·
1230 ·
1231 ·
                          170 JJ= KGMIN
MPD= MP2
1232 ·
1233 ·
1234 ·
1235 ·
                          MPD= MP2
MFLAG= 2
WRITF (6.174) MPD.JJ.GMIN

'74 FORMAT(/.' THE QUADRATIC PICKED THE WRONG CONSTRAINT.'./

1.' THE MOST INFEASIBLE CONSTRAINT IS TYPE '.I3.

2' NUMBER '.I4./.' WITH A VALUE CF'.F14.7)

DO 171 I= 1.IHP1

171 VI(I) = X(I) - ACC(3.I)

DO 175 I= 1.4

175 DOT(I) = 0.0

IE (MPD.F2.-1) GO TO 180
1236 .
1237 ·
1239 ·
1239 ·
1240 .
1241.
                                   DOT(I)= 0.0

IF (MPD.F2.-1) GO TO 180

DO 172 I= 1.KNU

IK= I+ KMU

DOT(1)= DOT(1)+ GI(JJ.I)*ACC(3.IK)

DOT(2)= DOT(2)+ GI(JJ.I)*VI(IK)
1242.
1243 .
1244 .
1246 .
                          1247 .
1248.
1247.
1251 .
1 252 .
125.1.
1254 .
1255.
1256 .
1257.
                           STP= (-88(JJ)-001(1))/DCT(2)
WRITE (6.185) G2(JJ.1).EE(JJ).DOT(1).DOT(2).STR
185 FORMAT(1X.' G2. 88. DOT1. DOT2. STR :'.5F13.6)
1250 .
```

```
1260.
1261.
1262.
1263.
                  1264 .
                  1265.
1265.
1268.
1270.
1271 .
1272.
1274 .
1275.
1276.
                         QUADS FINDS THE SMALLEST NUNNEGATIVE RUDT OF U(1)*ALPHA **2 + U(2)*ALPHA + U(3) = 0. IF THERE IS ONE. IF NOT, IMAG IS SET = 1.
: 278 .
1270.
1280.
1281 .
                          SUPROUTINE QUADS (U. IMAG. ALPHA, BETA)
                         GURROUTINE QUADS(U.IMAG.ALPHA.BETA)

IMPLICIT REAL## (A-H.O-Z)

INTEGER PO.PDI.PO2.S.R.SS.RR.ZFLAG.RS.P

INTEGER#2 JH(35C).DIGMA(95Z).KINBAS(130Z).IDBAS(130Z)

COMMONITALER/ TOLEZ.TOLBO.TGLCV.THETA.SIPMX.STPRD

COMMONIDIA/ IH.N.M.KMJ.KNU.MPI.NM.ITAIL

DIMENSION U(3)
1292.
1 284 .
1285.
1286 .
1287 .
                          IMAG= C
1288.
                         IMAGE C

IF (DABS(U(1)).GT. TOLFZ) GD TO 10

IF (DABS(U(2)).LT.TOLFZ) GC TO 8

ALPHA = -U(3)/U(2)

BETA= ALPHA
1200 .
1290.
 291 .
1292.
1203.
                          *FTU=N
1294 .
                      a IMAG= 1
1295.
                        RETURN
1296.
                    10 RT = U(2)**2 - 4*U(1)*U(3)
IF (FT) 20, 30, 40
1299 .
1299 .
                           THE QUADRATIC DUES NOT INTERSECT THIS CONSTRAINT.
1301.
1302.
1303.
1304.
                    20 IMAGE
RETURN
                         THERE IS A DOUBLE FOOT.
1305.
1306 ·
1307 ·
1308 ·
1309 ·
1310 ·
                    30 ALPHA = U(2)/(U(1)*2.)
                         SE TUEN
                           TWO STAL POOTS EXIST.
 311 .
                    40 DIS = DSGRT(RT)
ALPHA = (-U(2)-DIS)/(2.*U(1))
BETA = (-U(2)+DIS)/(2.*U(1))
IF (ALPHA .LE. BETA) RETURN
1312.
1314 .
1 315 .
 1316.
                          SAVE - ALPHA
1317.
                         ALPHA = META
1319.
                         GETURN
```

```
1320.
1321.
1322.
                                                 SUBROUTING DSENT(ALPHA.FNORM)

IMPLICIT REAL*3 (A-H.J-Z)

COMMON/NG*T/ H(10.11).X(10).Z(10).ACC(3.10).BLAM(10)

C)MMON/CEM/ H.N.M.KMU.KNU.MP1.NM.ITAIL

DIMENSION Y(10).F(10)

DO 5 I= 1.IH

Y(1)= X(I)- ALPHA*Z(I)

CALL FIN(F.Y)

CALL FIN(F.Y)

CALL NORM (F.S2.IH)

IF (50 .LT. PNORM) RETURN

ALPHA= ALPHA/2.

TE (ALPHA .GT. .00001) 3L TO 1

RETURN

END
                                C
1323.
1324.
1325.
1326.
 1327.
1329 .
1 329.
1330 .
1332.
1 334 .
1335.
1336.
1337.
                                                   SUBFRUITINE GTN(IFL.O.U.F)

IMPLICIT REAL *8 (A-H.)-Z)

INTEGE PD. PDI.PD2.S.R.35.RR.ZFLAG.RS.P

INTEGE *2 JH(350).DIGMA(952).KINEAS(1302).IDBAS(1302)

CDMMCN/NEWT/ H(10.11).X(10).Z(10).ACC(3.10).BLAM(10)

COMMCN/CIM/ IH.N.M.KMU.KNU.MPI.NM.ITAIL

COMMCN/SCAL/ ST.NB.JJ.MFLAG.IHPI.P.PD.MPD.KFUN.KJAC

CDMMCN/CIM/ IT.NB.JJ.MFLAG.IHPI.P.PD.MPD.KFUN.KJAC

CDMMCN/INCX/ JH.DIGMA.KINBAS.IDBAS

COMMCN/INCX/ JH.DIGMA.KINBAS.IDBAS

COMMCN/INCX/ NUH(10).MU(10).MU(10)

DIMENSION U(10).0(10).F(10)
1339.
    340 .
1341.
1344 .
1345 .
1346.
                                                  OIMENSION U(10).0(10).F(10)
Q(IHP))= 0.0
CALL FIN(F.U)
THETA= U(IHP1)
1 348 .
1 350 .
                                                   1F (KNU. 0.0) GO TC 21

DO 20 I = 1.KNU

IK= I+ KMU

QS= H(IK) + U(IK)

Q(IK)= THETA * DS - U(IK)
1351.
1352.
1353.
1 354 .
1355 .
                                          21 CONTINUE
21 IF (KMU. 0.0) GO TC 24
21 I= 1.KMU
12 MU(I)
1356.
1 358 .
1 359.
                                                                260 .
1361 .
1362.
1363 .
1 364 .
 1 355 .
1366.
1368.
1 350.
                                         Q(I) = THETA=35 - 51.

23 CONTINUE
24 IF (IFL.FQ. 0) RETURN

KFUNE KFUNE 1

SUM= 0.0

IF (IFL.FQ.2) GO TO 3)

OO 25 I= 1.IHP1

SUV= SUM + ACC(2.I) * U(I)
1371 .
1 375 .
    176 .
   377.
175.
1370 .
1 380 .
                                          25 CONTINUE
                                                   O(THE1) = SUM - AT
1381 .
1 392.
```

```
30 IF (MPD.=Q.-1) GO TO 50

IF (KNU.=Q.O) GO TC 45

DO 40 I= 1.KNU

IK= T+ KMU
1383.
1385 .
1386.
                        SUM = SUM+ G1(JJ.T) *L(IK)

40 CONTINUE

45 Q(IHP) = SUM+ BA(JJ)

RETURN
1388.
1 300 .
                        RETURN
50 IF (KMU.FQ.C) GD TO 60
20 55 I= 1.KMU
50M= 50M+ G2(JJ.I)*L(I)
55 CONTINUE
 301.
1393.
1395.
                        60 3([HF1) = SUM+ 88(JJ)
1396.
                             CNE
1708 .
1399.
                        SUBSOUTING DERIVG CALCULATES THE HESSIAN OF THE HETPACTION, PLUS SOME OTHER GRADIENT
1400.
                       SPECIFIED BY IFL.

IFLET: NO OTHER GRADIENT

IFLET: A SPHERE DETERMINES THE GRADIENT.

IFLET: ONE OF THE LINEAR CONSTRAINTS IS

THE GRADIENT.
 401.
                   C
1402.
                  0000
1404.
1406.
                             1409.
1410.
1411.
1411.
1413.
1414.
1415.
1416.
1417.
1418.
1419.
1420.
1421.
 423 ..
                       1424.
1426 .
1427.
  428.
                                     108= KINBAS (IP)
20 24 J= 1.KNU
JK= J+ KMU
 427.
1430 .
1431 .
  432.
                                            H(1.JK) = H(1.JK) - TMTH*G1(108.J)
                                     CONTINUE
30 TO 28
H(I.I) = H(I.I) - TMTH
1473.
1433.5
                        30 T) 24

H(1.1) = H(1.1) - TMTH

26 CONTINUE

30 35 I = 1.1H

IF(1 .GT. KMU) GO TO 30

H(I.1HP1) = F(I) + 3LAM(I)

GO TO 35

H(I.1HP1) = F(I) + U(I)

31 H(I.1HP1) = F(I) + U(I)
1434.
1436 .
 438 .
1439.
1441.
```

```
1451
1452
1453
1454
                                                                                                     SE TULN
                                                                                    1456
  459
  1441 .
  462
                                                                                    60 CONTINUE

25 TUPN

10 00 CO J= 1.[HP1

H(JHP1.J) = 0.0

IF (J.GT.K.W) GO TO 80

H(JHP1.J) = G2(JJ.J)
1464 .
  1 461 .
    467.
      46 9.
                                                                                     BO CONTINUE
 1476.
                                                                                                         FND
  1472
                                                                                                         SURF UT INE CONCHE (GMIN.KGMIN.MP2)
                                                                                    THIS CUBROUTING EVALUATES THE CONSTRAINT FUNCTIONS AND FINDS THE SMALLEST VALUE IN GMIN. ZFLAG IS BOTH AN INDUT AND OUTPUT PARAMATER. IF ZFLAG.EG.1 INITIALLY. THEN ONLY THE NONLINEAR CONSTRAINT IS EVALUATED. ZFLAG IS SET EQUAL TO 2 IF ANY CONSTRAINT IS NONPOSITIVE. OTHERWISE IT REMAINS ZERO.
 1474.
  1477 .
                                                                  0000
   1478.
      470 .
 1480
1481
1482
1487
                                                                                                        IMPLITIT REALTH (A-H-C-Z)

PEAL MA ((800)
INTEGER PD -PD1 -PD2 - 3 - R - SS - RR - ZFLAG - RS - P
INTEGER M2 (350) - DIGMA (952) - KINBAS (1302) - IDBAS (1302)
INTEGER M2 ISTYPE - LA - LE - IA - IE - PUN - L (20) - IC (800)
COMMON/NEW T/ H (10 - 11) - X (10) - X (10) - ACC (3 - 10) - BLAM (10)
COMMON/NEW T/ H (10 - 11) - X (10) - E (10) - E (10) - C (1
     484 .
  1485.
1486.
14H7.
   1444.
      499.
   1490.
 1492 .
                                                                                                         COMMON/DIM/ IH.N.M.KMU.KRU.MPI.NM.ITAIL

GM IN = 10.

10 = 1 + M

TK = JH(!)

GG = 0.0

IF (KNU.50.0) GD TC 15

01 10 J = 1.KNU

JK = J + KMU

GG = GG + G1(!.J)*X(JK)

CONTINUE

GG = GG + BA(!)

XX(IK) = GG

IF (IK.FO.M .OR. IK.LE.IH) GD TU 20

IF (MFLAG.=20.3 .ANL. IK.LU.NM) GD TU 20

IF (GG.GF.GMIN) G) TU 20

GMIN = GG
   1494 .
    405.
 1496.
   1498.
   1499.
1500
1501
1502
1503
                                                                                      15
1504
1505
1506
1507
                                                                                                                                     641 N= GG
                                                                                                                                     KGMIN= 1
```

```
1511.
1512.
1513.
                                              20 CONTINUE

20 40 1= 1.N

IK= DIGMA(1)
                                                                       1K= 71Gmm(1)

UG= 0.0

1F (KMU,EQ.0) GQ fC 35

DO 30 J= 1.KMU

GG= GG+ G2([.J)*X(J)
1514.
1515.
1516.
1517.
                                                                    CONTINUE
1519.
                                              30
                                                                      GG= GG+ BB(I)
P((K)= GG
IF (IK .EQ. M) GD TC 40
IF (GG.GE.GMIN) G) TC 42
1519.
1521.
1522.
1523.
                                                                        MIN= GG
                                                                        K GM TN=
1525.
1526.
1527.
                                                                        MD2 = -1
                                               40 CONTINUE
1328 .
                                                         END
   329 .
 1530.
                                                     SUBSTRUCTINE FTN EVALUATES THE FUNCTION F(X) = \langle B(MU) + D1 \times X(MU) + D1 \times Y(MU) \rangle \times \langle F1 \times X(MU) \rangle \times \langle B(MU) + D2 \times X(MU) + CIAG(X(NU)) \rangle \times \langle F2 \times X(MU) + F3 \rangle
                                   0000
1531.
1533.
                                                         SUBFOUT INE FIN(F,Y)
 1535.
                                              THIS RUUTINE EVALUATES THE FUNCTION WHICH THE ENDPOINT SUPPOUTINE IS CURRENTLY TRYING TO FIND A ROOT OF. IF MFLAG TOULLS I THEN THE LAST FUNCTIONAL COMPRISING F IS ONE OF THE INEQUALITY CONSTRAINTS—— THE INEQUALITY DETERMINED BY THE PARAMETERS MPD AND JJ.
1534 .
1537.
1534 .
1540.
1541.
1542.
1543.
                                                        IMPLICIT REAL*8 (A-H-)-2)

PEAL*4 C(800)

INT'GTE PO,PDI.PD2.5.R.35.RR.ZFLAG.RS.P

INT'GTE*2 JH(350),DIGMA(952).KINEAS(13J2).IDBAS(1302)

INT'GTE*2 JSTYPE.LA.LE.IA.IE.PUN.LC(20).IC(800)

COMMON/NEWT/ H(10.11).X(10).Z(10).ACC(3.10).BLAM(10)

COMMON/RECST/BF(1C).BF2(10).E1(10).E2(10).C.IC.LC

COMMON/RECST/DI(10.10).D2(9.10).F1(9.10).F2(9.10)

COMMON/SCAL/ BT.NR.JJ.MFLAG.IHP1.P.PD.MPD.KFUN.KJAC

COMMON/DIAL* TH.N.M.KMU.KNU.MP1.NM.ITAIL

DIMENSION F(IH).Y(IH)

KMU1= KMU+ 1
   544 .
 1545.
1546 .
1548 .
1549 .
   550.
1551.
1552.
1553.
                                                         MU1= KMU+ 1
KFUN= KFUN+ 1H
IF (KMU,=Q.0) GO TO 23
DO 20 I= 1.KMU
DOT= 3.0
1 554 .
 1555.
   556.
1557.
                                             DOT= 0.0

IF (KNU.ED.0) GD TO 15

DO 10 J= 1.KNU

JK= J+ KMU

10 DOT= DOT+ F1(I.J)*Y(JK)

15 F(I)= -Y(I)*(DOT+ F1(I))+ BF1(I)

16 KMU.ED.0) GD TO 20

DO 10 J= 1.KMU

18 F(I)= F(I)+ D1(I.J)*Y(J)

20 CONTINUE

27 IF (KNU.ED.0) RETURN

DO 40 I= KMU1.IH

TMB= I- KMU
1559.
   561 .
 1562.
1563.
1364 .
 1565 .
 1566 .
1567.
1568 •
1569 •
1570 •
```

```
DOT= 0.0

IF(KMU.EQ.0) GO TO 30

DO 25 J= 1.KMU

25 DOT= DOT+ F2(IMB.J)*Y(J)

30 F(I)= -Y(I)*(DOT+E2(IMB))+ 3F2(IMB)

IF (KMU.EQ.0) GO TO 40

DO 32 J= 1.KMU

32 F(I)= F(I)+ D2(IMB.J)*Y(J)
1571 •
1572 •
1573 •
1574 •
 1575 .
1576.
      574 .
      579.
                                                                                AN CONTINUE
      580 .
                                                                                                RETUCN
 1581.
                                                                                                 END
                                                                                                  SURFOUTINE DERIV
      393 .
                                                                                          THIS SUBFOUTINE CALCULATES THE EXACT JACUBIAN OF THE BILLINEAR FUNCTION DEFINED BY BLCCHS. THERE MAY OR MAY NOT BE ANOTHER FUNCTIONAL APPENDED WHICH WE ARE ATTEMPTING TO MAKE BINDING WITH NEWTON'S METHOD.
      584 .
 1585.
1586.
1587.
1586 .
                                                                                            IMPLICIT PEAL *8 (A-H, D-Z)

9EAL ** C(300)
INTEGTR PD.PD1.PD2.S.R.SS.RR.ZFLAG.RS.P
INTEGTR* DJ.PD1.PD2.S.R.SS.RR.ZFLAG.RS.P
INTEGTR* Z JH(350).PIGMA(952).KINHAS(1302).IDBAS(1302)
INTEGTR* Z JH(350).PIGMA(952).KINHAS(1302).IC(800)
COMMON/NEWT/ H(10.11).X(10).Z(10).ACC(3.10).BLAM(10)
COMMON/NEWT/ H(10.11).X(10).Z(10).ACC(3.10).BLAM(10)
COMMON/RLCSTZ/PD1(10.10).D2(9.10).F1(9.10).F2(9.10)
COMMON/RLCSTZ/PD1(10.10).D2(9.10).F1(9.10).F2(9.10).F1(9.10).F2(9.10)
COMMON/RLCSTZ/PD1(10.10).F2(10).F2(10).F2(10).F2(10).F1(9.10).F2(9.10)
COMMON/RLCSTZ/PD1(10.10).F2(10).F1(9.10).F2(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1(9.10).F1
      549.
                                                                                                   IMPLICIT PEAL#8 (A-H. 3-Z)
      590 .
1591 .
1563.
      594 .
      595.
 1596 .
  1597.
      598 .
  1599.
   1600.
1602.
1603.
  1504.
 1505
                                                                                              1604 .
  1507.
  1510 .
1611.
1512.
1513.
1614.
  1615.
1615
1617
1617
1617
1627
                                                                               20
                                                                                                                        H(I,J)= H(I,J)- DDT- EI(!)

CINTINUE

IF (KNU.EQ.O) GO TO 50

DO 40 != KMU1.IH

IMP = I - KMU

H(I,J)= D2(IMB.J)- F2(IMB.J)*X(I)

CONTINUE
                                                                                 30
1621.
1622.
1623.
1624.
                                                                                50 CONTINUE
                                                                               1525 .
  1527.
1529.
1629.
1630.
```

```
1631.
1632.
1633.
1634.
                                                                                                                     H(I,J)= - X(I)*F1(I,JMU)
                                                                       50 CONTINUE
70 CONTINUE
70 CONTINUE
70 IF (*NU.EQ.O) RETURN
DO 80 I= 1.KNU
IK= KMU+I
IF (*MU.EQ.O) GO TO 77
DO 75 K= 1.KMU
75 H(IK.IK)= H(IK.IK)- F2(I.K)*X(K)
77 H(IK.IK)= H(IK.IK)- E2(I)
80 TONTINUE
RETURN
END
HORM(Y.S1.N)
 1635.
  1636.
 1637 •
1638 •
1639 •
 1640 .
1641.
1542.
1643.
1544.
                                                                                           FND

SUBROUTINE NORM(Y.S1.N)

IMPLICIT REAL*8(A-H.O-Z)

DIMENSION Y(10)

S1= 0.0

DD 10 I = 1.N

S1= S1+ Y(I)*Y(I)

S1= CSCKT(S1)
1645 •
1646 •
1647 •
1649 •
 1540.
  1650 .
 1651 •
1652 •
1653 •
                                                                                              RETURN
                                                                                             FND
SUPPOUTINE DECOMP(NN.4.UL)
IMPLICIT REAL *8(A-H.D-Z)
COMMUNITY IPS(30).KDET.KCUNT.ISING.KEND
DIMENSION A(12.17).UL(10.10).SCALES(10)
1654 .
1656 ·
1657 ·
1659 ·
                                                                                             NE NN
KDET = 1
                                                                                      ISING= 0
100 S I= 1.N
1PS(I) = I
4000NRM= 0.0
2 J= 1.N
1UL(I.J) = A(I.J)
IF (ROWNRM-DAJS(UL(I.J))) 1.2.2
POWNRM= DAPS(UL(I.J))
POWNRM= DAPS(UL(I.J))
THE RESULT OF THE RES
1659 .
 166C .
                                                                                   INIT! ALTZE IPS.UL. AND SCALES.
 1661.
 1563.
 1564 .
 1665.
 1566 .
 1667.
  1668.
1570.
 1571 .
1672.
                                                                                 4 CALL SING(1)
ISING= 1
SCALES(I)= 0.
5 CONTINUE
1574 · 1575 · 1576 ·
1677.
1678.
1670.
                                                                                 GAUSSIAN ELIMINATION WITH FARTIAL PIVOTING
                                                                                             MMI= N- 1
DD 17 K= 1.NM1
PIG= 0.0
DO 11 I= K.N
 1581 .
 1582 .
                                                                                                                                          10= IPS(I)
SIZE= DAJS(UL(IP,K))*SCALES(IP)
  1583.
     584 .
1585 •
1585 •
1557 •
                                                                                                                                                           (SIZE-BIG) 11.11,10

PIG= SIZE

10XPIV= 1
                                                                            10
 1690.
                                                                            11
                                                                                                                    CONTINUE
                                                                                                                   IF (ATG) 13.12.13
CALL SING(2)
 1589 .
 1590.
                                                                            12
```

```
ISING= 1
GO TO 17

IF (IDXPIV-K) 14.15.14

J= IPS(K)
IPS(K)= IPS(IDXPIV)
IPS(IDXPIV)= J
KD.T= -KOET

KO= IPS(K)
PIVOT= UL(KP.K)
KP1= K+1
DO 16 I= KP1.N
ID= IPS(I)
EM= -UL(IP.K)/PIVOT
UL(IP.K)= -EM
DO 15 J= KP1.N
UL(IP.J)= UL(IP.J)+ EM*UL(KP.J)
CONTINUE
1691 •
1692 •
1693 •
1694 •
1696 •
                                                                                                           13
   1697.
   1590 .
                                                                                                           15
  1500.
  1701 .
   1702.
  1704.
 1706.
                                                                                                                                                                 CONTINUE
                                                                                                       16 CONTINUE

17 CONTINUE

KP= IPS(N)

IF (UL(K2.N)) 19.18.19

18 CALL SING(2)

ISING= 1

19 RETURN
 1709.
1709.
1710.
1711.
1711.
1714.
1715.
1716.
1717.
                                                                                                                                 FNO
                                                                                                                                 SUBROUTINE SOLVE(NN.UL.E.X)
IMPLICIT REAL **B(A-H.O-Z)
COMMONZINTZ IPS(30).KDET.KCUNT.ISING.KEND
DIMENSION UL(10.10).**E(10).X(10)
   1718.
 1719.
1720.
1721.
1722.
1723.
                                                                                                                                   V= NN
                                                                                                               V= NN

NP1= N+ 1

IP= IPS(1)

X(1)= B(IP)

DO 2 I= 2.N

IM1= I-1

SUM= C.7

TO 1 J= 1.IM1

1 SUM= 5UM+ UL(IP.J)*X(J)

2 X(I)= B(IP)- SUM
   1724 .
   1725.
 1726.
  1720 .
 1729
1730
1731
1732
1733
                                                                                                      C
1734 · 1735 · 1736 · 1737 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 1739 · 17
 1739.
                                                                                 C
 1741.
         743.
  1744 .
  1746 .
  1747.
  1744 .
                                                                                                                   4 X(1)= (X(1)-SUM)/DIV
  1740.
                                                                                                                                 RETURN
   175C.
                                                                                                                                  END
```

```
1751.
1752.
1753.
                                                          SUBROUTINE SING(IWHY)

11 FORMAT(1X. "MATRIX WITH ZERC RGW IN DECOMPOSE.")

12 FORMAT(1X. "SINGULAR MATRIX IN DECOMPOSE. ZERO DIVIDE IN SOLVE")

15 (IWHY- 1) 1.1.2

1 WRITE (6.11)

GO TO 10

2 WRITE (6.12)

10 RETURN

END
 1754.
 1755 .
   756 .
1757.
 1750
  761 .
                                                                        END
                                                                      END
SURPOUTINE DEGUG(MCDE)
IMPLICIT REAL*8 (A-H.O-Z)
REAL*9 MIN
REAL*9 TIME
INTEGER PD.PD1.PD2.S.R.SS.RR.ZFLAG.RS.P
INTEGER*2 JH(350).DIGMA(952).KINBAS(1302).IDBAS(1302)
INTEGER*2 JH(350).DIGMA(952).KINBAS(1302).IC(80C)
DOUBLE PRECISION E(3000)
REAL A(4000).C(30C).CMIN.COND.ERMAX.SUMINE
 1762.
1763 .
 1764 .
 1765.
 1766 .
 768 .
 1760.
1770 .
   771.
                                                                   COMMON DSUM.DPRJD.CY,DE.DP.B(350).X(350).Y(350).YTEMP(350).
14.F.CMIN.COND.ERMAX.SUMINF.ICNAM(1302.2).NAME(20).
2NTEMP(20).KINP.ITIM.JTIM.ITINV.JTINV.MSTAT.IDBJ.IROWP.IVIN.IVOUT.
31TCNT.INVFRQ.ITRLIM.IFFFZ.JCOLP.NECW.NCOL.NELEM.NETA.NLELEM.NLETA.
1772.
 1774 .
                                                       ZNITEMP(201-KINP-ITIM.JTIM.ITINV.JTINV.MSTAT.IOBJ.IRO)

ZITCNT.INVFRO.ITRLIM.IFFZ.JCOLP.NKCW.NCOL.NELEM.NETA

4NGALEM.NINF.NUELEMM.NUETA.NNEGDJ.NLINES.ISTYPE(35C).

ELA(13C2).LE(2002).FUN(3).

EIPUNC.NOEGI.NDUAL.NIPIW.IFBAS.IFCRSH

COMMON 1A(40C0).IE(3000)

COMMON/LPI/PI(1302).XX(1302)

COMMON/ZLCST/BFI(10).BF2(10).EI(10).E2(10).C.IC.LC

COMMON/ZLCST/BFI(10).BF2(10).FI(9.10).F2(9.10)

COMMON/ZNCONS/GI(350.10).G2(40C.IC).BA(350).BB(400)

COMMON/INDXZ/ JH.DIGMA.KINBAS.IC3AS

COMMON/INDXZ/ JH.DIGMA.KINBAS.IC3AS

COMMON/SCAL/ BT.NB.JJ.MFLAG.IHPI.P.PD.MPD.KFUN.KJAC

COMMON/SCAL/ BT.NB.JJ.MFLAG.IHPI.P.PD.MPD.KFUN.KJAC

COMMON/CIM/ TH.N.M.KMU.KNU.MPI.NM.ITAIL

IF (MODE-2) 10.30.60

1 TIME= 1.0

WRITE(6.20) TIME

20 TOPMAT (1X.* THE TIME LEFT IS NOW *.F9.6.* SEC.*)

RETURN

30 WRITE (6.40)
 775.
   776
    778 .
    779.
 1780.
 1781 •
   784 .
 1785 .
 1786 .
 1787.
     788.
 1789 .
1791.
     703.
    794.
1795 •
1796 •
1797 •
                                                          RETURN

RETURN

RETTE (6.40)

RETTE (6.41) (BA(I).I= 1.M)

RETTE (6.49) (BB(I).I=1.N)

IF (KNU.50.0) 60 TC 35

RETTE (6.42) ((GI(I.J).I= 1.M).J= 1.KNU)

35 IF (KMU.50.0) RETURN

RETTE (6.43) ((G2(I.J).I= 1.N).J= 1.KMU)
     798 .
1799.
1900.
1901.
1902.
                                                        WRITE (6.43) ((G2(I.J).I= 1.N).J= 1.

WETURN

4C FORMAT (/.IX.* LINEAR CCNSTRAINTS.*)

42 FORMAT (IX.* G1= *.7F13.5)

41 FORMAT (IX.* BA= *.7F13.5)

43 FORMAT (IX.* BB= *.7F13.5)

43 FORMAT (IX.* G2= *.7F13.5)

66 IF (MCDF + 4) 65.7C.10C

67 WRITE (6.85) (JH(I).I= 1.NEOW)

RO FURMAT (IX.* KINBAS= *.2014)
     404 .
 1905.
    907.
 I Ang.
 1 310 .
```

```
## PORMAT (1X, JHE '.2014)

RETURN

RE
1813.
1814.
1915.
1916.
                                                                                                        85 FORMA" (1X." JH= ".2014)
RETURN
70 WRITE (6.71) KNU.KMU
   1314.
 1919.
   1921 .
   1222.
   1923.
 1824 .
   1826 .
    1929.
 1829.
  1931 .
  1832.
  1934 .
   1936 .
   1337.
   1838.
        839 .
 1940.
  1941 .
 1342.
   1343.
   1844 .
  1345.
   946 .
  1947.
   1948.
        949 .
   1950.
1952.
   1854.
   1855.
  1956.
   1958.
    1959.
    1960.
   1861 .
   1862.
   1963.
                                                                                                                                  END
```

H

References

- [1] Elken, T.R., "On The Solution of Nonlinear Equations by Path Methods," TR SOL 77-25, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, October 1977.
- [2] Elken, T.R., "The Computation of Economic Equilibria by Path Methods," TR SOL 77-26, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, October 1977.
- [3] Forsythe, G.E., The Solution of Linear Systems of Equations, New York: Prentice Hall.
- [4] Tomlin, J.A., "Users Guide for LPM1," unpublished report, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, 1975.
- [5] Tomlin, J.A., "Users Guide to LCPL -- A Program for Solving Linear Complementarity Problems by Lemke's Method," TR SOL 76-16, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, August 1976.
- [6] Tomlin, J.A., "Programmer's Guide to LCPL -- A Program for Solving Linear Complementarity Problem's by Lemke's Method," TR SOL 76-25, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, October 1976.
- [7] Wilson, R., "The Bilinear Complementarity Problem and Competitive Equilibria of Linear Economic Models," TR SOL 76-2, Systems Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, January 1976.

(14) REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
SOL-78-17	NO. 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Sublitle)	S. TIPE OF REPORT & PERIOD COVE
BCA and HRA: Two Programs for Computing Econor	Technical Repert
Equilibria.	PERFORMING ORG. REPORT/NUMBE
	SOL 78-17
7. AUTHOR(e)	S. CONTRACT OR GRANT NUMBER(*)
Thomas Elken	NØØØ14-75-C-Ø267
	Ey-76-5-03-0326
9. PERFORMING ORGANIZATION NAME AND ADDRESS	AREA & WORK UNIT NUMBERS
Department of Operations Research SOLV Stanford University	NR-047-143
Stanford, CA 94305	NR-047-143
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Operations Research Program ONR	// August 2078
Department of the Navy, 800 N. Quincy Street Arlington, VA 22217	13. NUMBER OF PAGE
14. MONITORING AGENCY NAME & ADDRESS II different from Controlling Offi	
00 100	UNCLASSIFIED
1320.	SE DECLASSIFICATION/DOWNGRADIN
	15. DECLASSIFICATION/DOWNGRADIN
This document has been approved for public releases the distribution is unlimited.	
its distribution is unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different	
its distribution is unlimited.	
its distribution is unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different	nt from Report)
its distribution is unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different in Supplementary notes 18. Supplementary notes 19. KEY WORDS (Continue on reverse elde if necessary and identity by block number in the supplementary	nt from Report) mbor) action Algorithm
its distribution is unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if differently supplementary notes 18. Supplementary notes 19. KEY WORDS (Continue on reverse elde if necessary and identify by block number of the supplementary in the supplementary	nt from Report) mbor) action Algorithm
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different in Supplementary notes 19. KEY WORDS (Continue on reverse elde if necessary and identify by block number in the supplementary in the supplem	mber) action Algorithm g Methods
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different in Supplementary notes 19. KEY WORDS (Continue on reverse elde if necessary and identify by block not be in the abstract programming and identify by block not be in the abstract (Continue on reverse elde if necessary and identify by block not be in the abstract (Continue on reverse elde if necessary and identify by block not this report gives programmers information useful fication of BCA and HRA, two programs for comparisons.	mber) action Algorithm g Methods al for the utilization and monthing economic equilibria.
its distribution is unlimited. 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if differently supplementary notes 19. KEY WORDS (Continue on reverse elde if necessary and identity by block notes are programming and path-Following Bilinear Complementarity 10. ABSTRACT (Continue on reverse elde if necessary and identity by block manner. This report gives programmers information useful.)	mber) action Algorithm g Methods al for the utilization and months acting economic equilibria. To gorithm and HRA a homotopy a subroutines and variables to

DD 1 JAN 73 1473 EDITION OF ! NOV 68 IS OBSOLETE UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (Then Det

8 765

